Optimal File-Distribution in Heterogeneous and Asymmetric Storage Networks

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Distributed Storage is Ubiquitous

Motivation

- Storing files on multiple machines is a common habit
  - P2P overlays basically represent storage networks
  - File-hosting services like Amazon S3, Rapidshare, etc.

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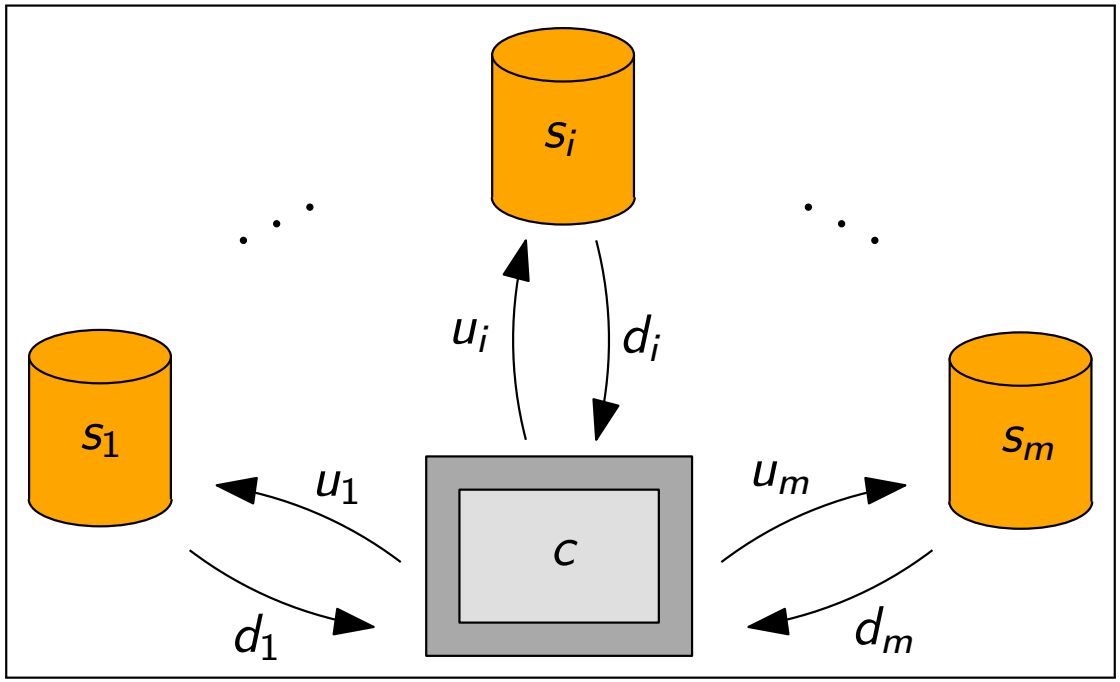
- Most existing approaches specifically tailored for data centers with homogeneous and symmetric network connections.
- Little attention to asymmetric bandwidths typical for end-user connections

Synopsis

Distribution Network

Problem Setting

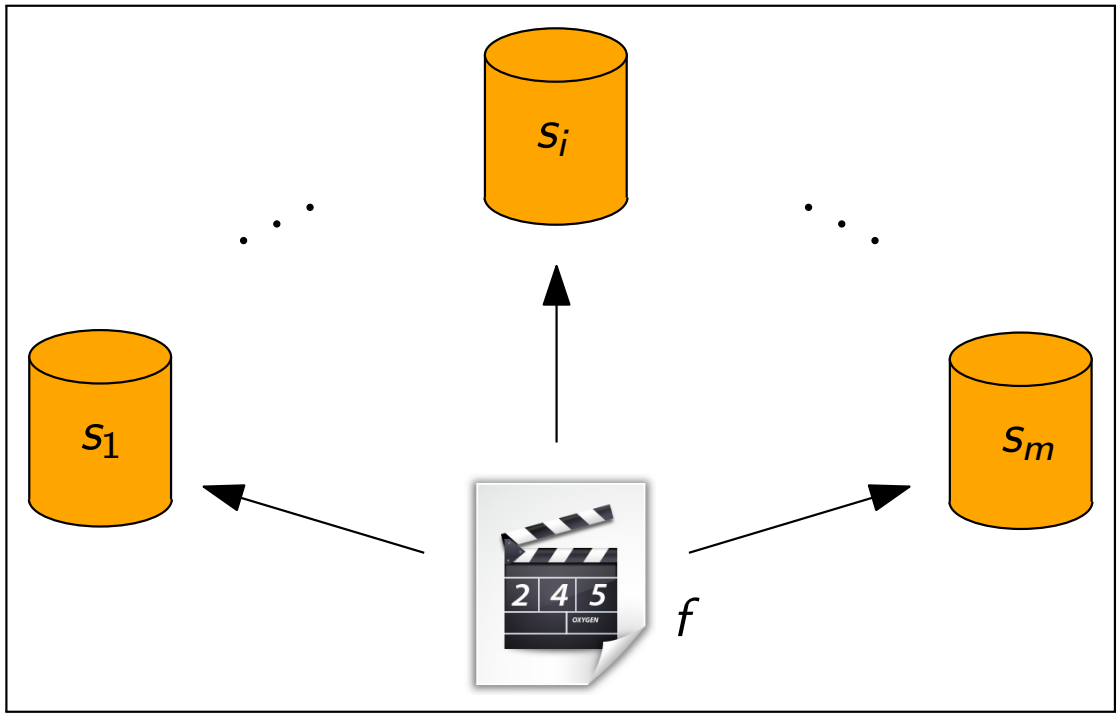


- Servers s_i with $i \in S = \{1, \ldots, m\}$, and one client c
- Each server s_i is characterized by its upload bandwidth u_i and download bandwidth d_i

Distribution Problem

Problem Setting

- How do we split a file f of size $|f|$ into fragments for the servers to minimize the time for one upload and n downloads?



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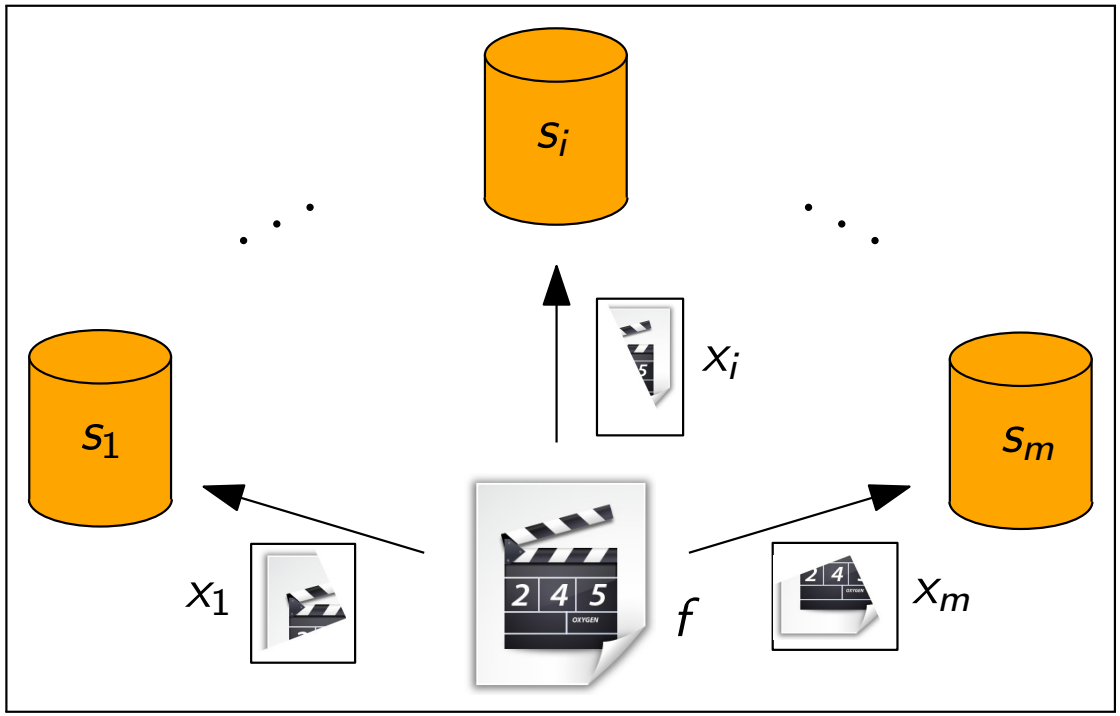

- Distribution vector $\mathbf{x} = (x_1, \ldots, x_m)$ with $\sum x_i = |f|$.
  x_i is the size of the fragment of server s_i.

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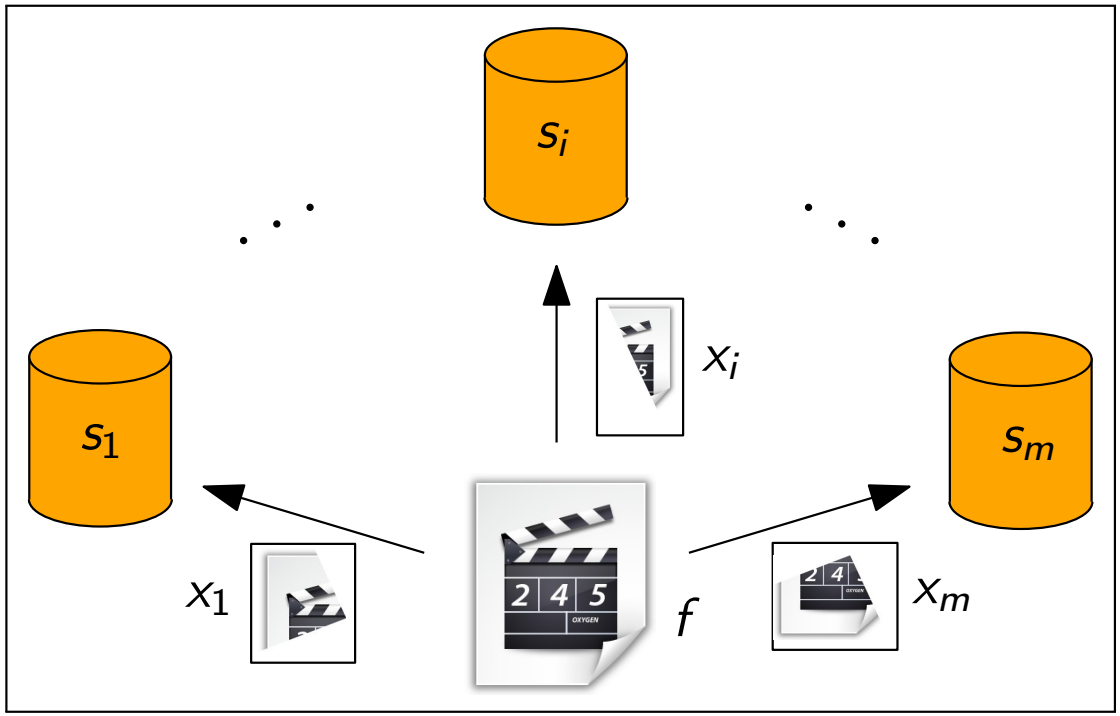

- Distribution vector $\mathbf{x} = (x_1, \ldots, x_m)$ with $\sum x_i = |f|$. x_i is the size of the fragment of server s_i.
- Equivalent simplified problem with $|f| = n = 1$.

Transfer Times

Problem Setting

The quality of a distribution $\mathbf{x}$ is given by the transfer times:

- Upload time:

$$t_u(\mathbf{x}) = \max_{i \in S} \left\{ \frac{x_i}{u_i} \right\}$$

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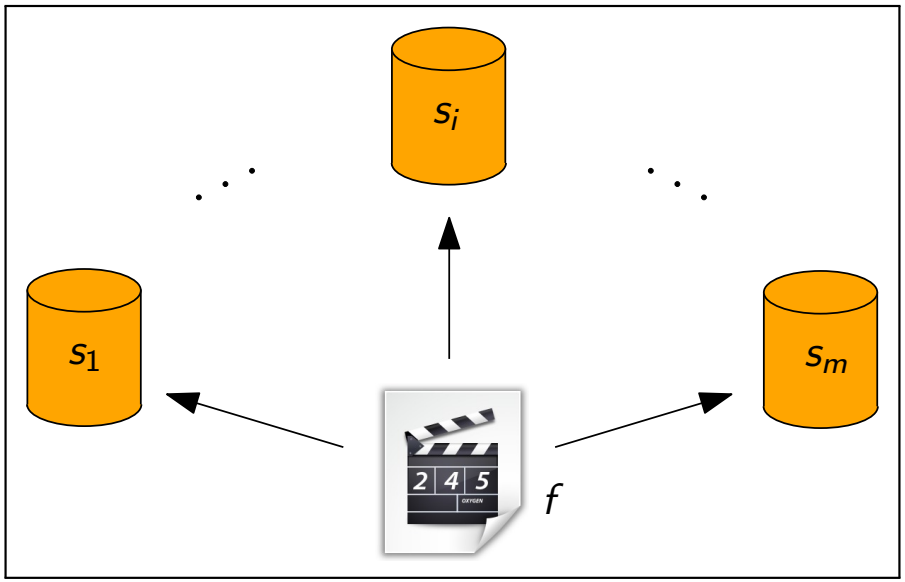
- Total time:

$$\mathrm{val}(\mathbf{x}) = t_u(\mathbf{x}) + (n \cdot)\, t_d(\mathbf{x})$$

Objective

Problem Setting

We are given a distribution network and a file f with size 1.



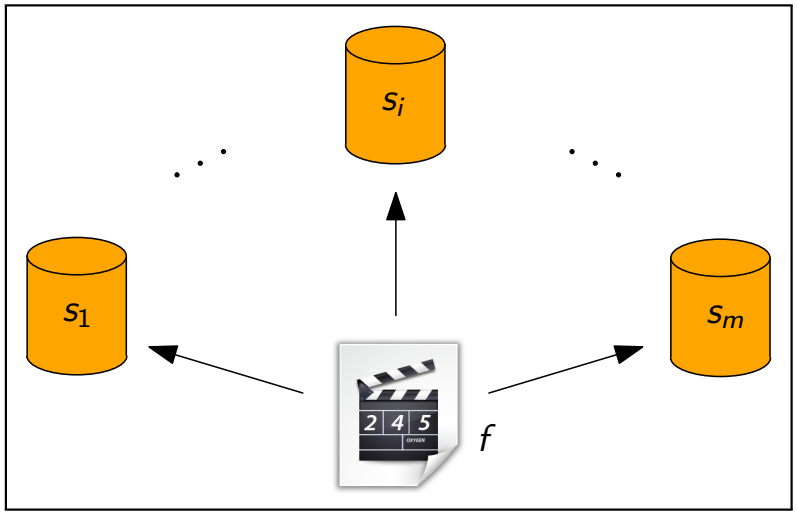
Objective

Find the optimal distribution $\mathbf{x}^*$ that minimizes the total time

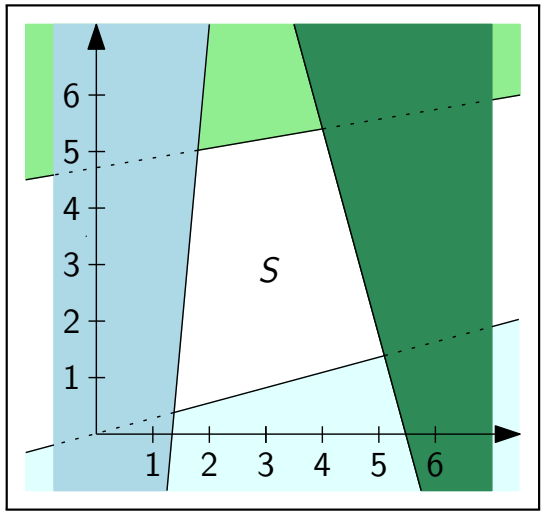
$$\text{val}(\mathbf{x}) = t_u(\mathbf{x}) + t_d(\mathbf{x}) \ .$$

Important Properties

Problem Setting



$\Rightarrow$



- The distribution problem can be formulated as linear program.
  $\Rightarrow$ Its solution space is a convex region.
  $\Rightarrow$ Every local minimum is global.

Non-Linear Optimization Problem

Linear Program

Upload/download bandwidths u_1 to u_m and d_1 to d_m

File distribution $x = (x_1, \ldots, x_m)$

Non-Linear Program

$$\text{minimize} \quad \max\left\{\frac{x_1}{u_1}, \frac{x_2}{u_2}, \ldots, \frac{x_m}{u_m}\right\} + \max\left\{\frac{x_1}{d_1}, \frac{x_2}{d_2}, \ldots, \frac{x_m}{d_m}\right\}$$

$$\text{subject to} \quad \sum_{i=1}^{m} x_i = 1$$

$$x_i \geq 0 \quad \text{for all } i \in \{1, \ldots, m\}$$

Linear Optimization Problem

Linear Program

Linear Program

$$
\begin{aligned}
\text{minimize} \quad & t_u + t_d \\
\text{subject to} \quad & \sum_{i=1}^{m} x_i = 1 \\
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Dimension: $m + 2$

Synopsis

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Maximum Flow in Distribution Problems

- Assume we are given upload time t_u and download time t_d

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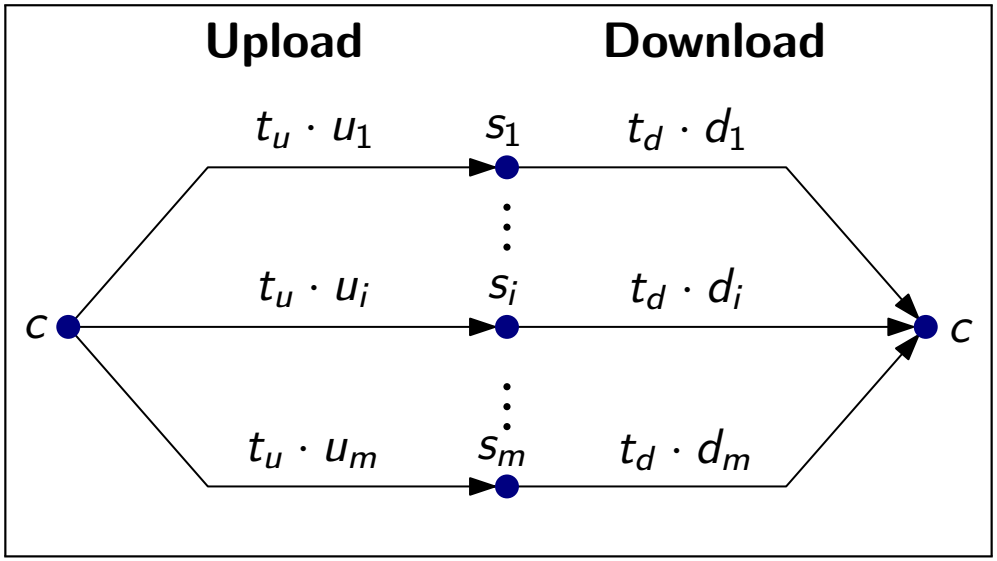

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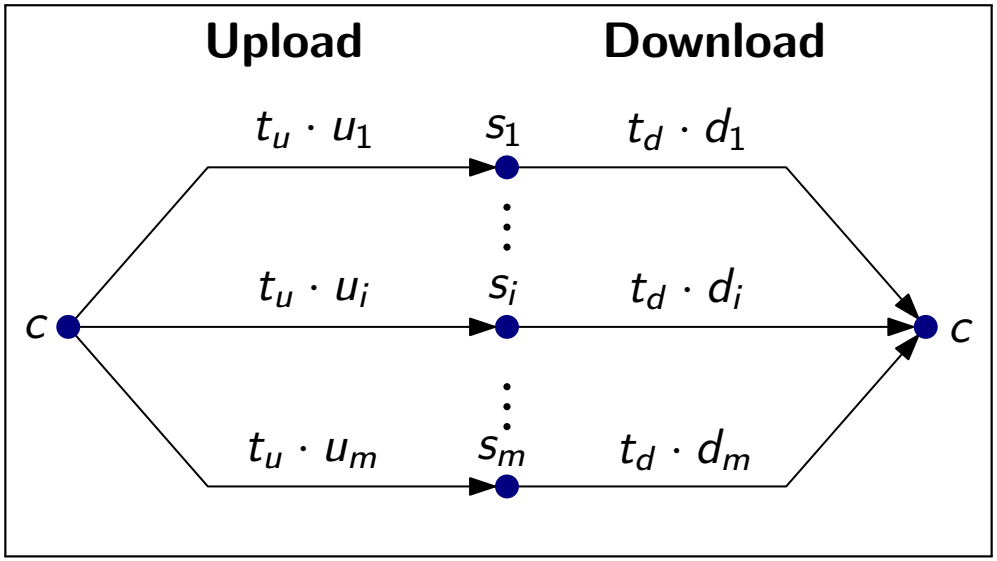

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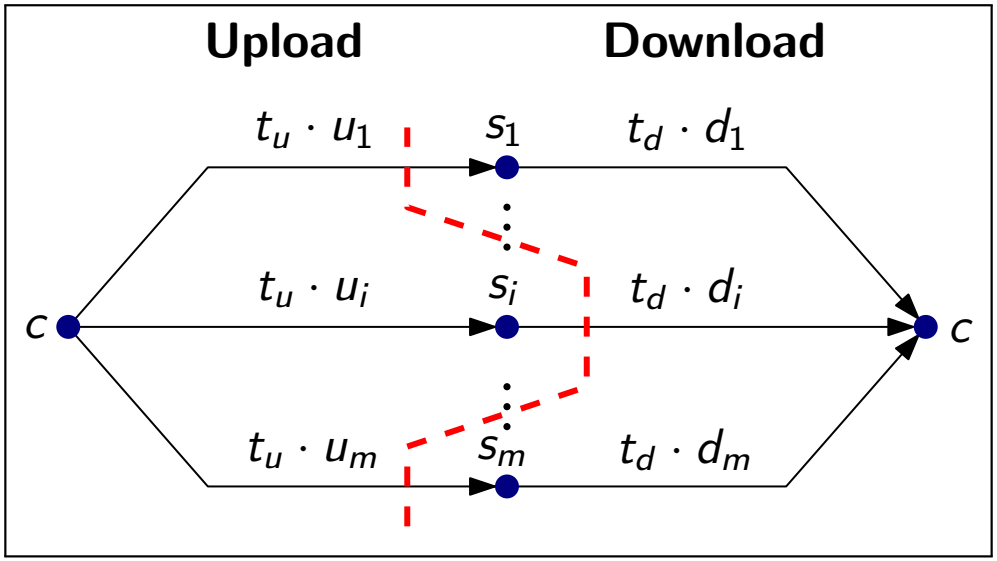

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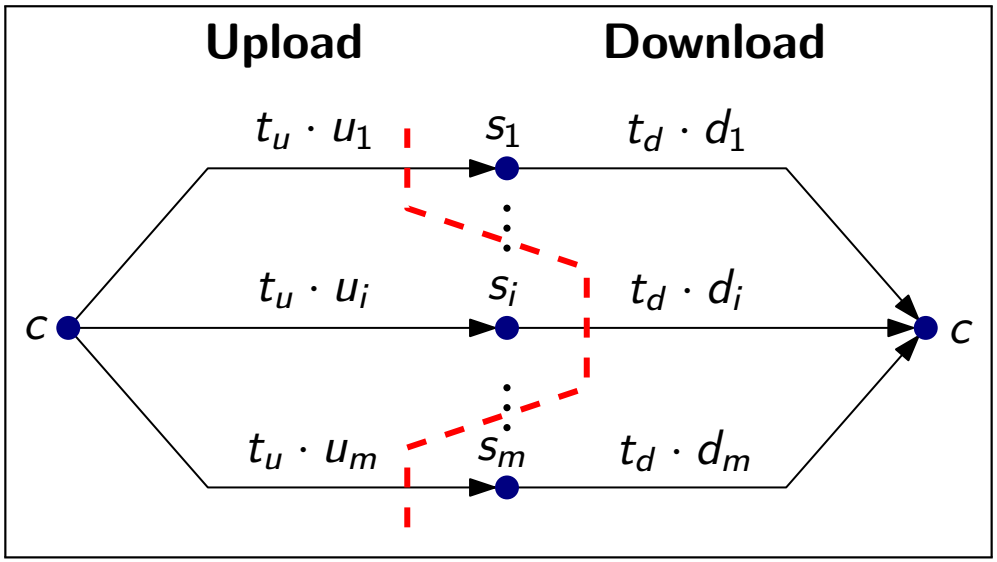

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$$\text{val}(f^*) = \sum_{i=1}^{m} \min\{t_u\, u_i,\ t_d\, d_i\}$$

Decision Predicate

Maximum Flow in Distribution Problems

- There exists a distribution such that f with $|f| = 1$ can be uploaded and downloaded in time t_u and t_d, respectively, if

$$\sum_{i=1}^{m} \min\{t_u\, u_i,\ t_d\, d_i\} \geq 1$$

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- We want to find the minimum values for t_u and t_d for which the above predicate holds.
- Substitute t_d by $T - t_u$ to obtain the total data function.

$$\delta_T(t_u) = \sum_{i=1}^{m} \min\{t_u\, u_i,\ (T - t_u) \cdot d_i\}$$

Total Data Function

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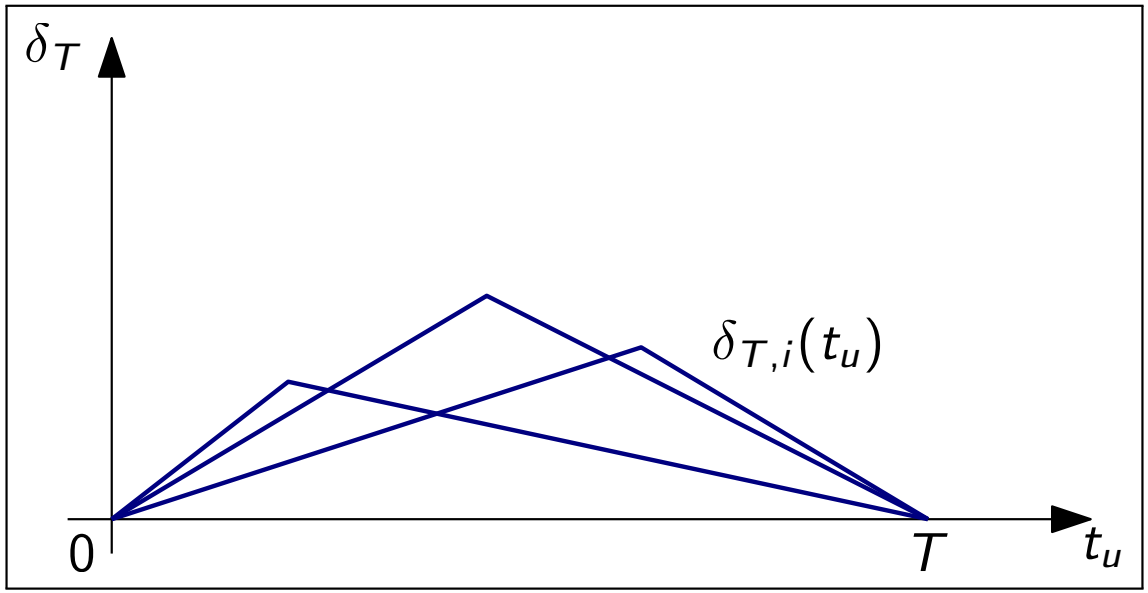

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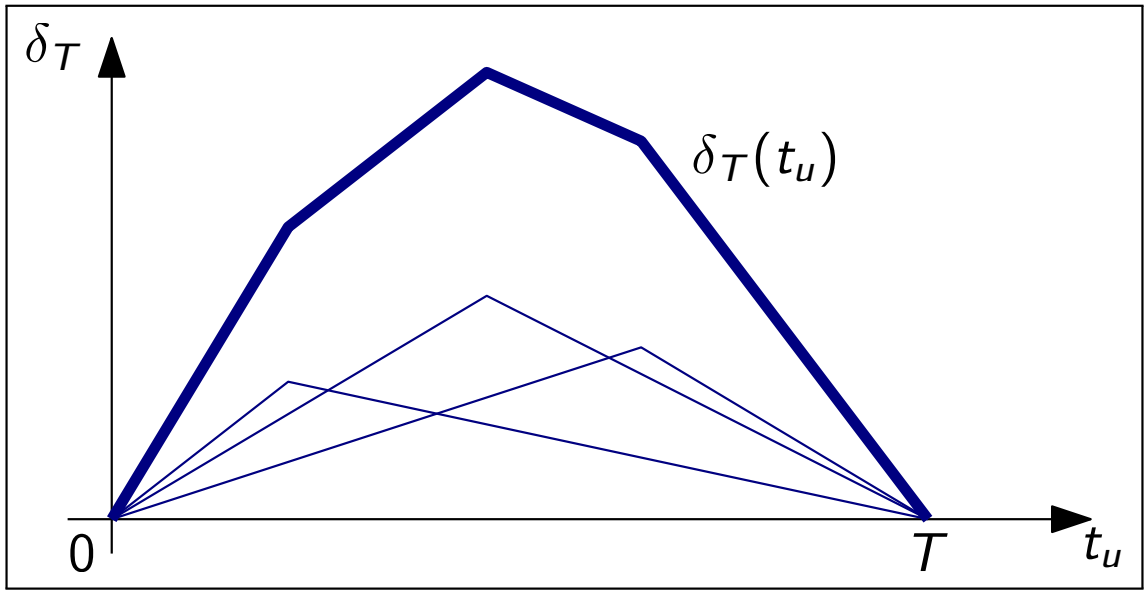

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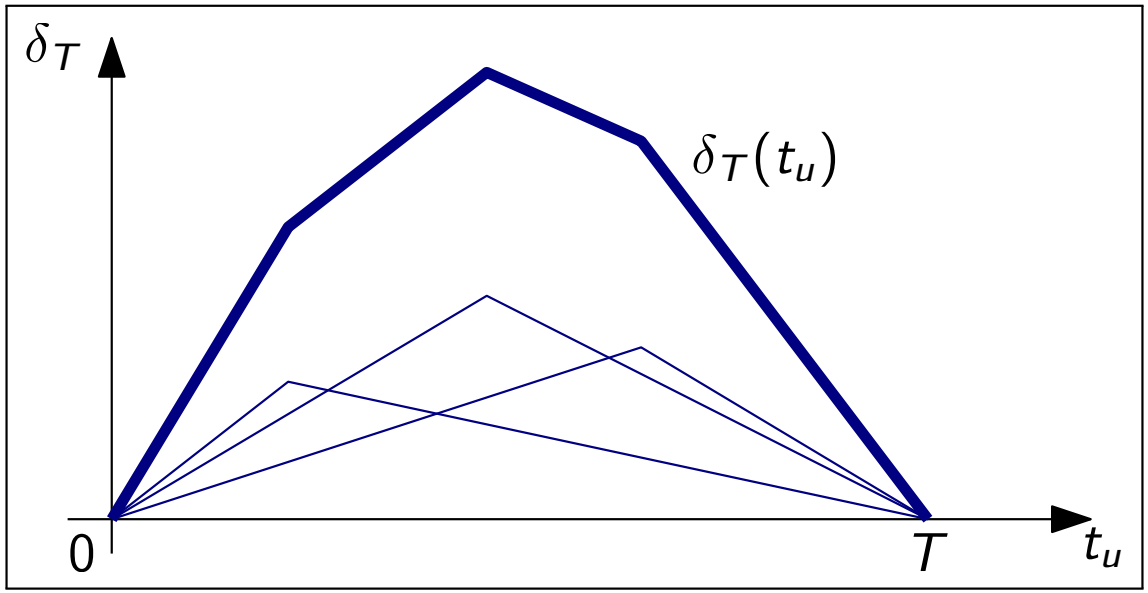
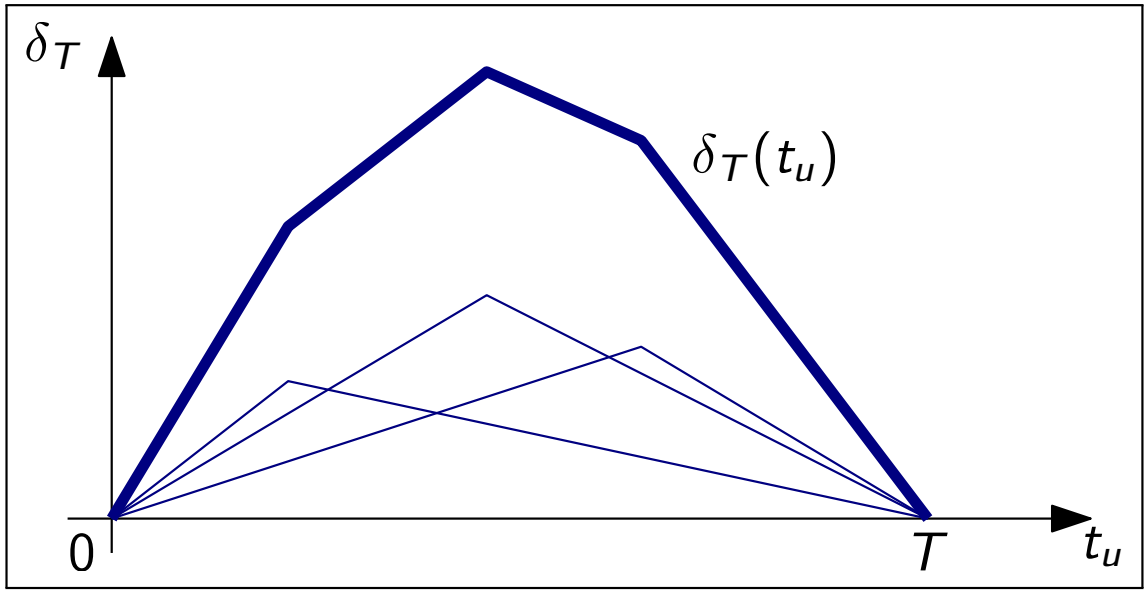


- Can be evaluated in $\mathcal{O}(m \log m)$ steps

Evaluation

The Total Data Function



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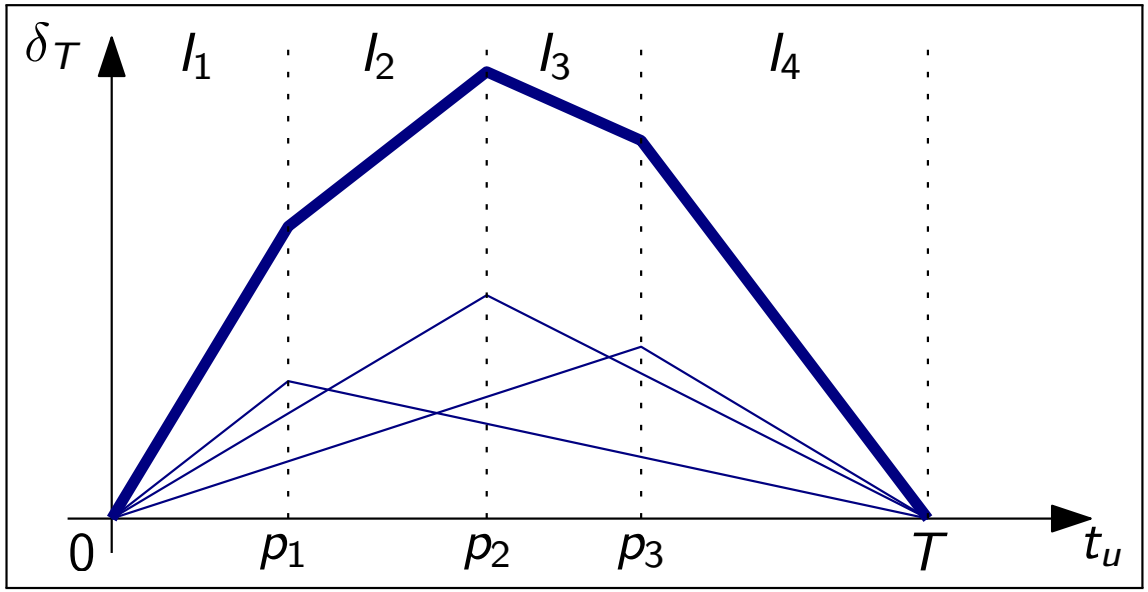

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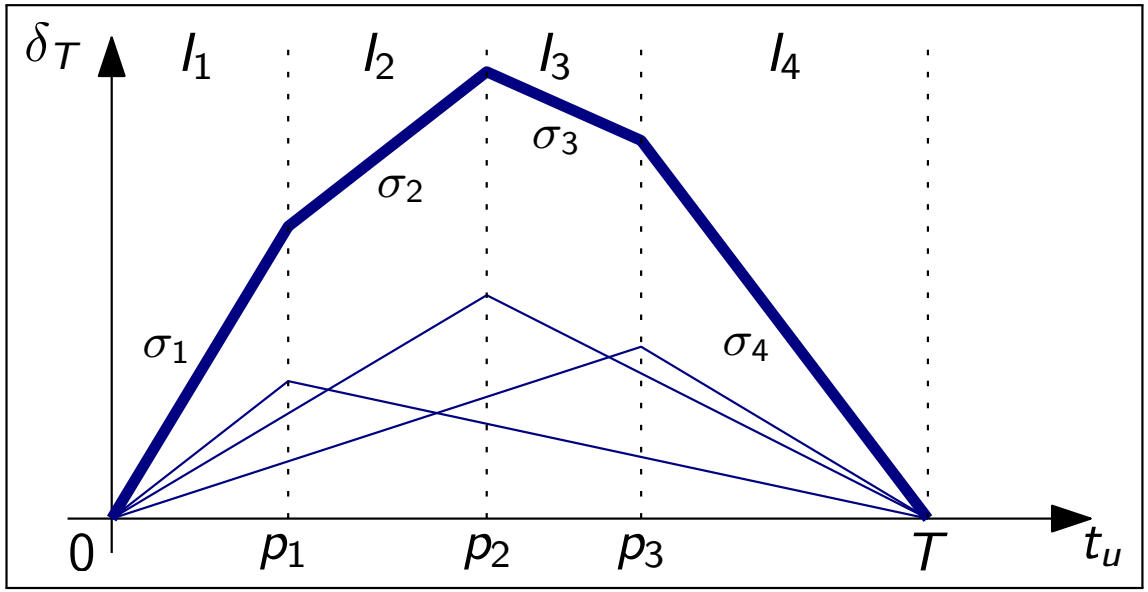

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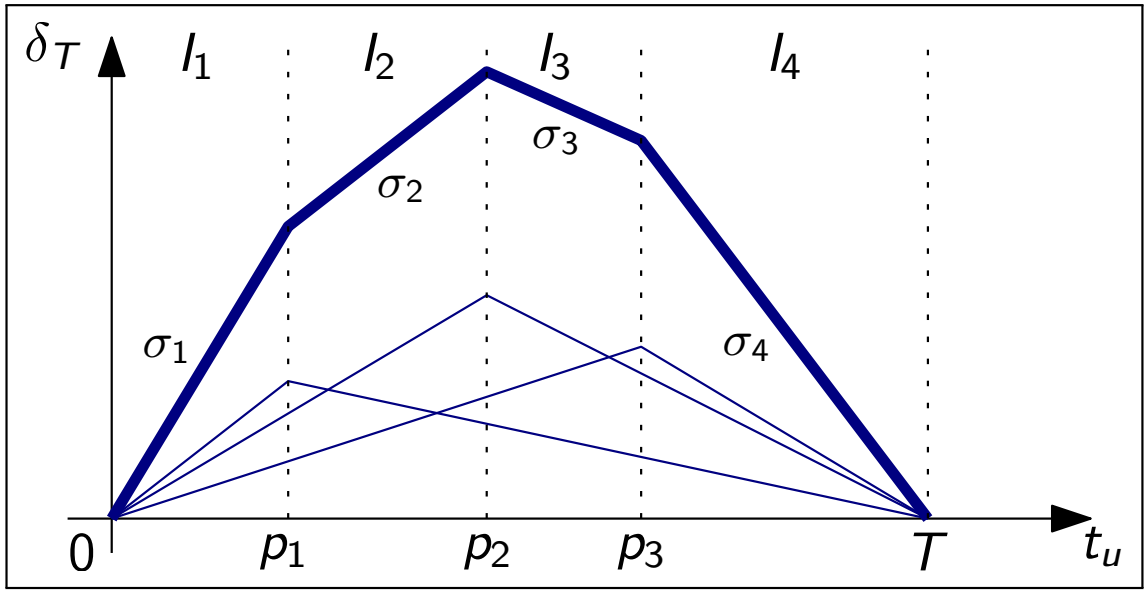

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Recursion formula

$$\delta_T(p_i) = \delta_T(p_{i-1}) + \sigma_i \cdot (p_i - p_{i-1})$$

Scaling the Total Data Function

Total Data Function in 3D

- The total data function $\delta_T(t_u)$ determines how much data can be uploaded to and downloaded again from the servers within time T for different values of t_u.

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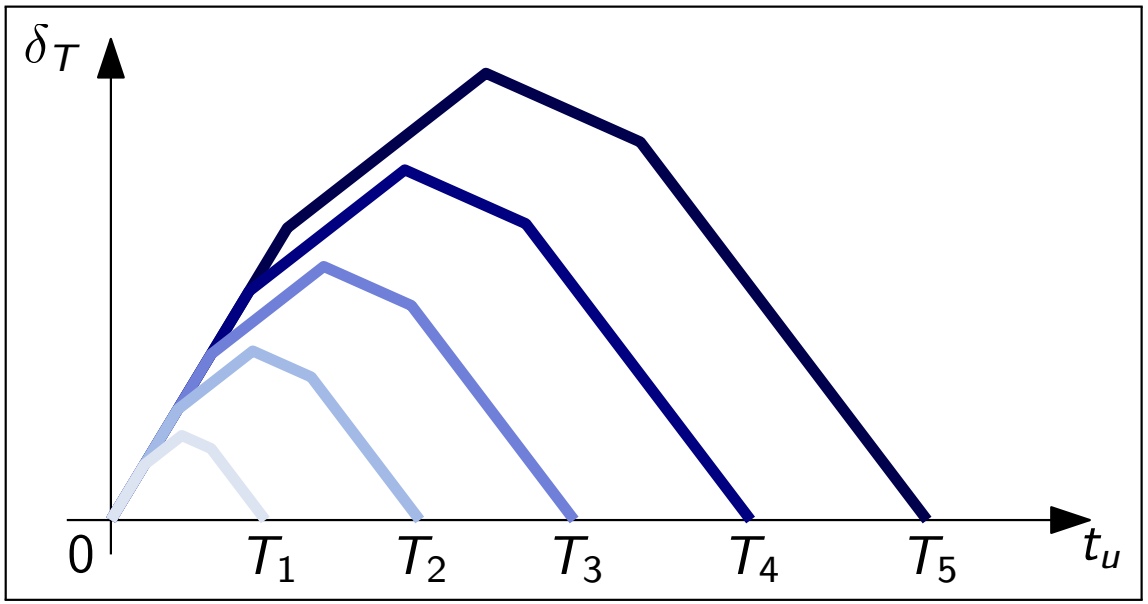

Two-Variable Total Data Function

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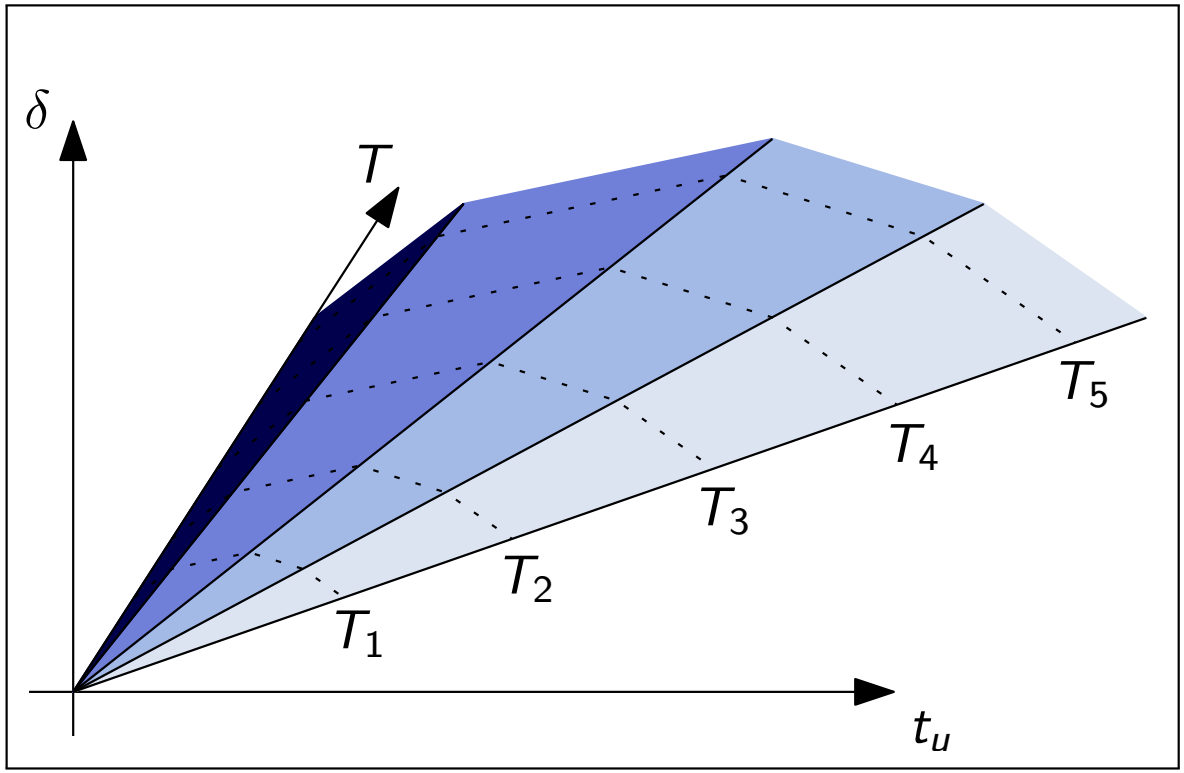
- Interpret $\delta_T(t_u)$ as two-variable function $\delta(T, t_u)$ now



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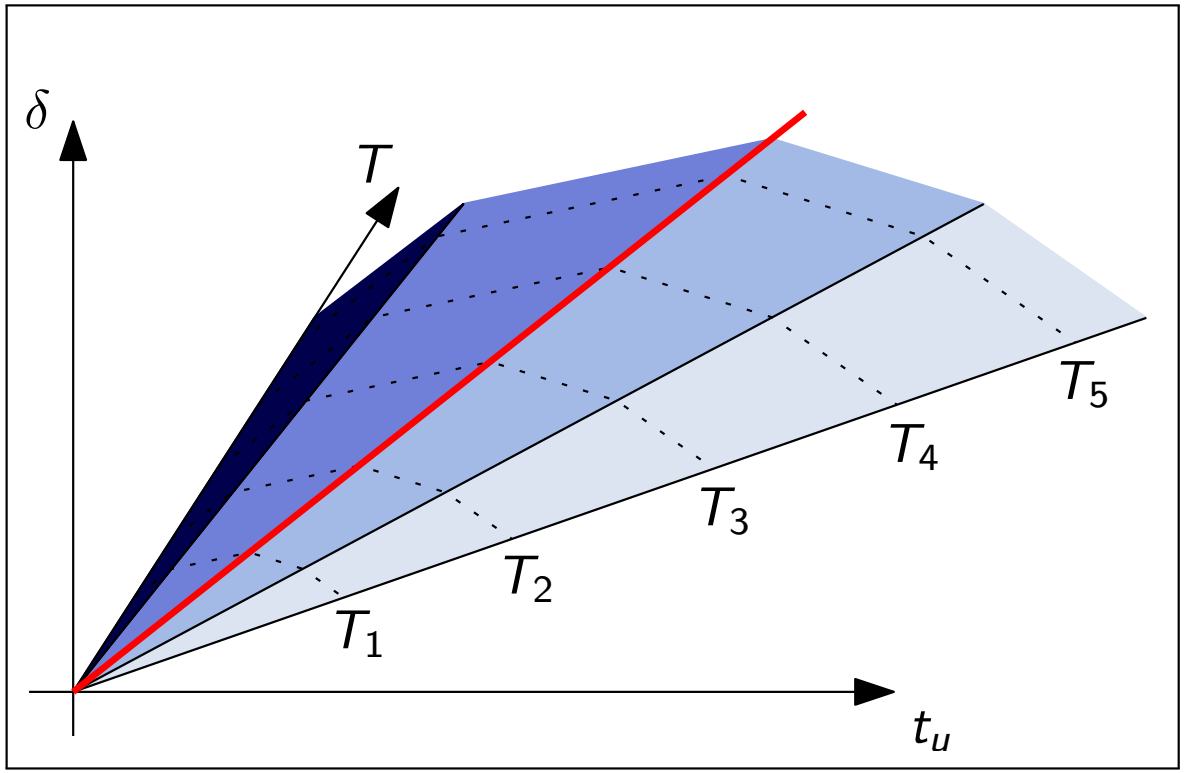

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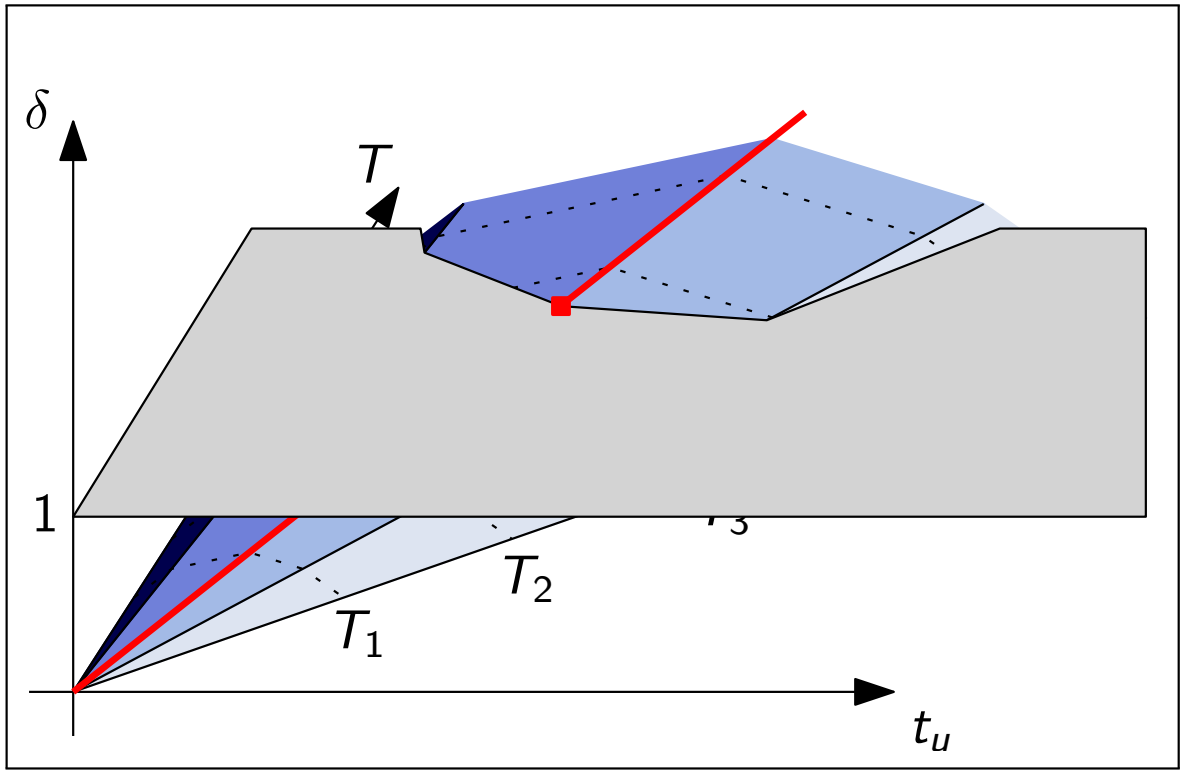
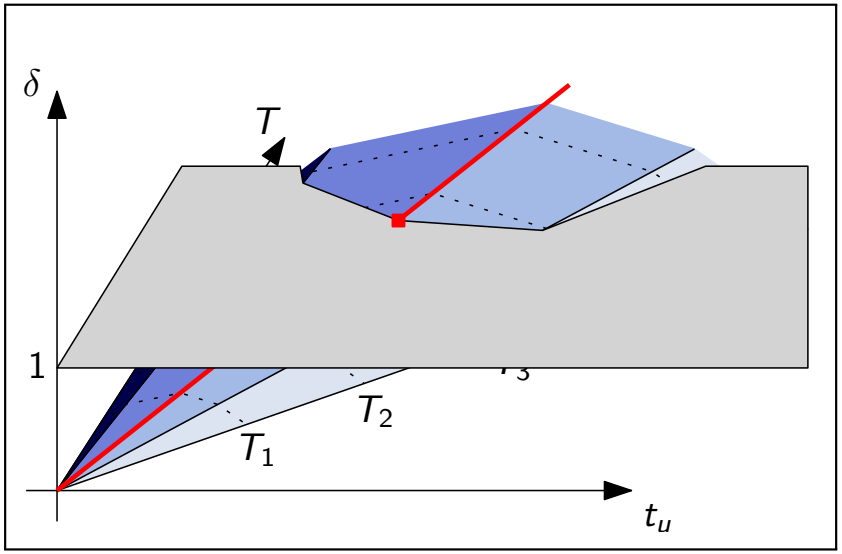


- To find the minimal value of T with $\delta(T, t_u) \geq 1$ for some t_u, establish straight line through the highest point of $\delta(T, t_u)$
- Determine where it intersects the plane $\delta = 1$

Determining the Actual Distribution

The Algorithm

- We determine the minimal value of T as depicted.



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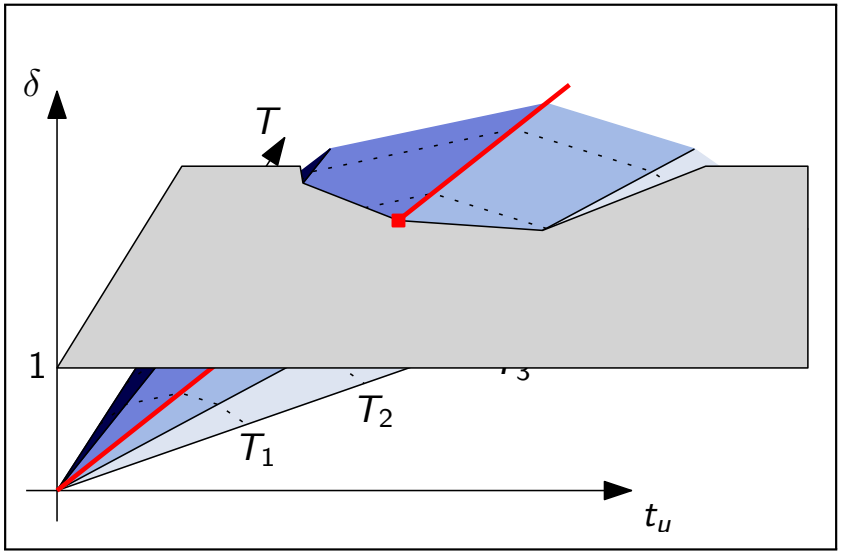

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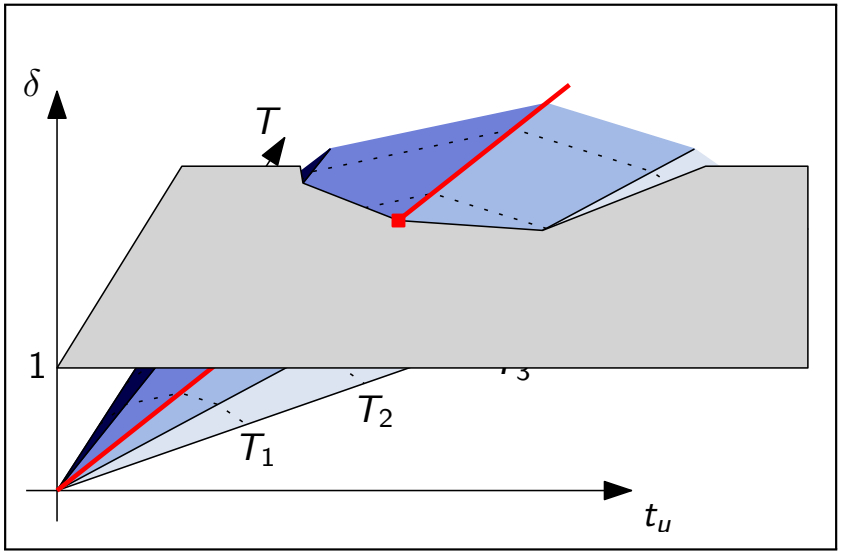

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- Algorithm to determine a solution with these time bounds:
  - Iterate through all servers $s_i \in S$
  - Assign to s_i the data amount $x_i = \min\{t_u\, u_i,\ t_d\, d_i\}$
  - Return the resulting distribution

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  - Iterate through all servers $s_i \in S$
  - Assign to s_i the data amount $x_i = \min\{t_u\, u_i,\ t_d\, d_i\}$
  - Return the resulting distribution
- Through the feasibility predicate

$$\sum_{i=1}^{m} \min\{t_u\, u_i,\ t_d\, d_i\} = 1$$

we know that $\sum x_i = 1$ and thus $\mathbf{x}$ is a valid distribution.

Runtime Complexity

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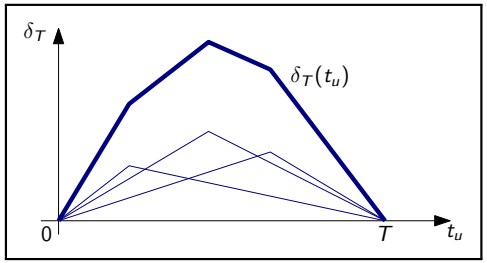
- Evaluating the total data function for a fixed value of T runs in $\mathcal{O}(m \log m)$ steps.



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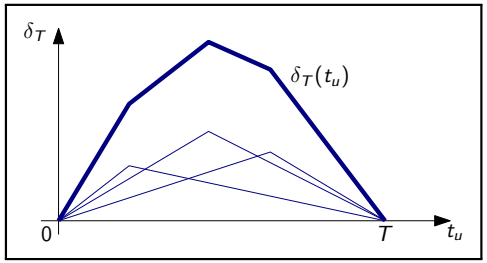
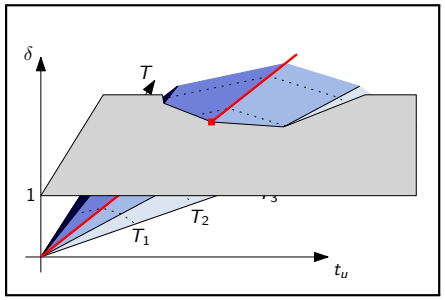


- Intersecting the straight line with the plane $\delta = 1$ is possible in constant time.



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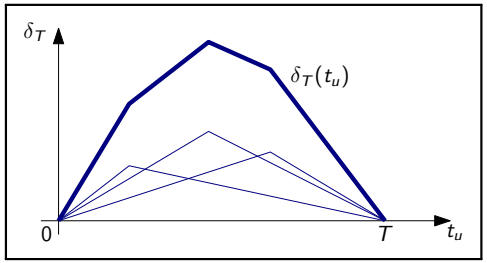

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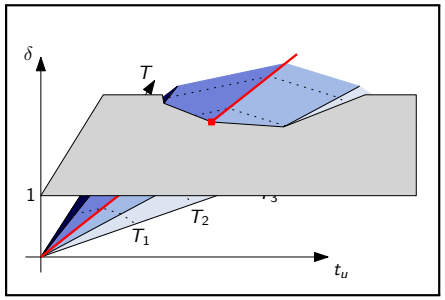

Runtime complexity: $\mathcal{O}(m \log m)$

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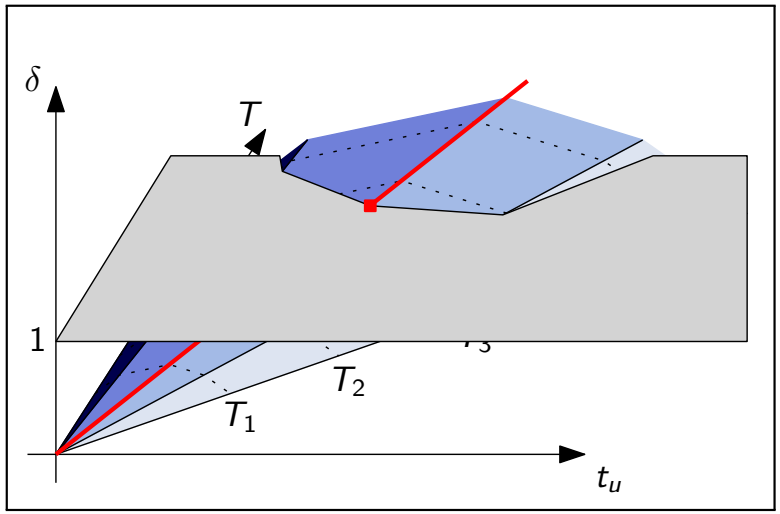

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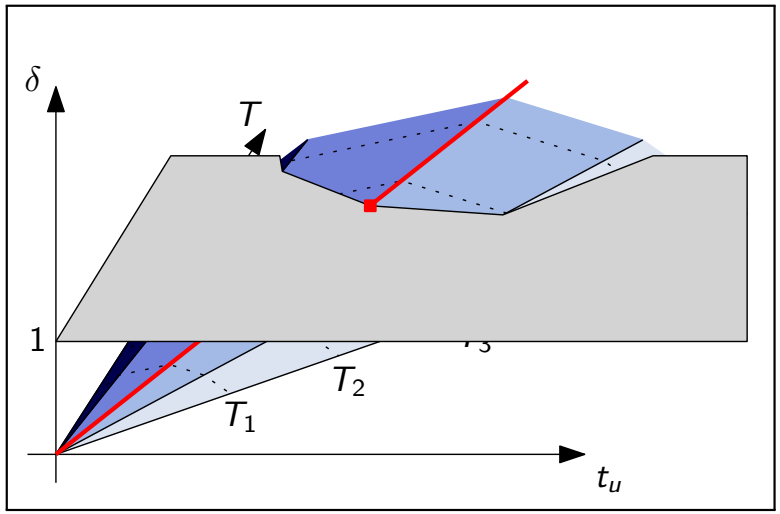
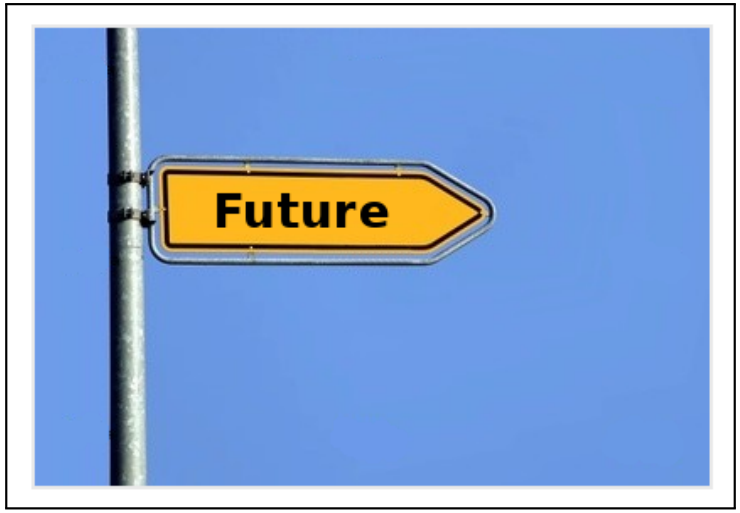
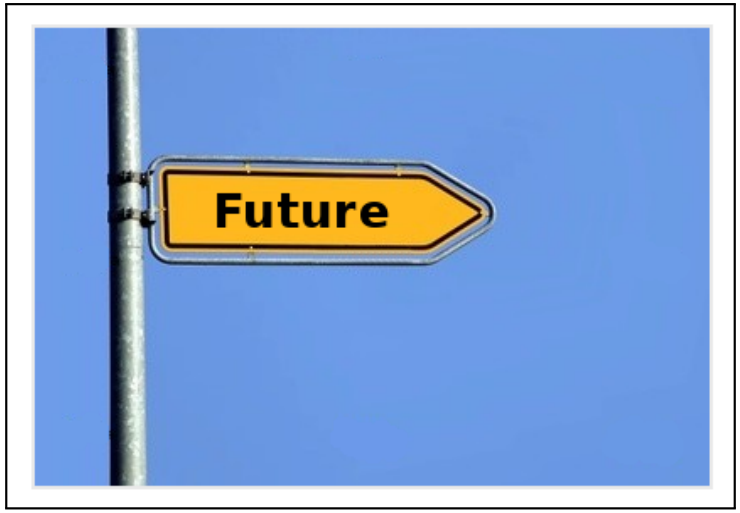


Future Directions



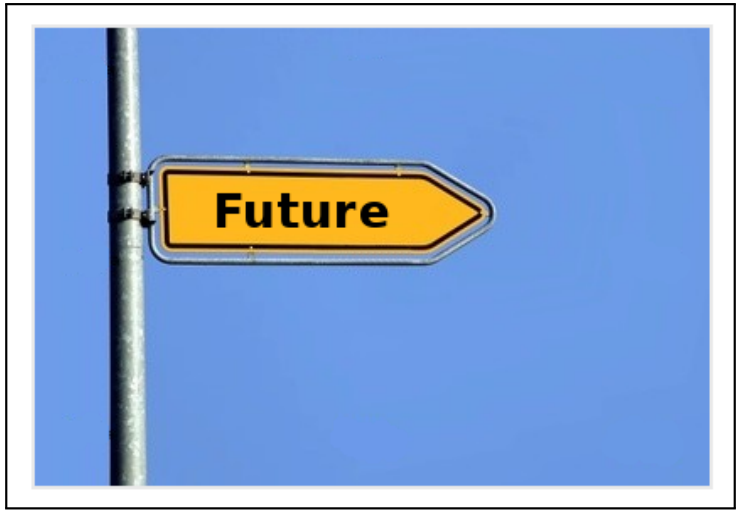
- Implementation of our ideas into real-world applications

Future Directions



- Implementation of our ideas into real-world applications
- Cope with non-static, fluctuating bandwidths

Future Directions



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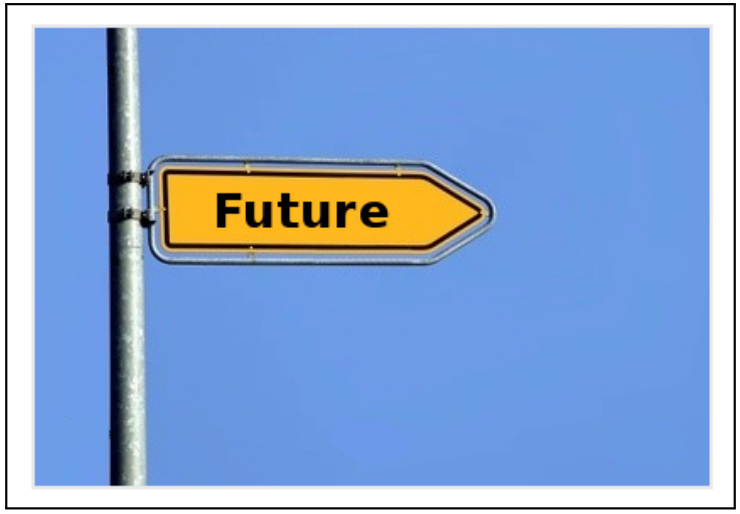

- Implementation of our ideas into real-world applications
- Cope with non-static, fluctuating bandwidths
- Respect limited storage capacity of servers
- Incorporation of redundancy to allow for failure tolerance

The End

Thank you for your attention!

Questions?