

Rapid Almost-Complete Broadcasting in Faulty Networks[★]

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Abstract

This paper studies the problem of broadcasting in synchronous point-to-point networks, where one initiator owns a piece of information that has to be transmitted to all other vertices as fast as possible. The model of fractional dynamic faults with threshold is considered: in every step either a fixed number $c(G) - 1$, where $c(G)$ is the edge connectivity of the communication graph, or a fraction α of sent messages can be lost depending on which quantity is larger.

As the main result we show that in complete graphs and hypercubes it is possible to inform all but a constant number of vertices, exhibiting only a logarithmic slowdown, i.e. in time $O(D \log n)$ where D is the diameter of the network and n is the number of vertices.

Moreover, for complete graphs under some additional conditions (sense of direction, or $\alpha < 0.55$) the remaining constant number of vertices can be informed in the same time, i.e. $O(\log n)$.

1 Introduction

Fault tolerance has been a crucial issue in distributed computing since its beginnings [3, 5, 6, 10, 16, 24]. Because a typical distributed system is designed to contain a large number of individual components, attention must be paid to the fact that, even if the failure probability of a single component is negligible, the probability that some components fail may be high. There are numerous ways how to cope with failures, using either probabilistic or deterministic approaches. In the probabilistic setting, it is supposed that a failure probability

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of each component follows some probability distribution [4, 8, 11, 25, 26]. Failures of individual components are usually assumed to be independent random events. The goal is to design algorithms and protocols that perform well with high probability if the failures follow the conjectured distribution.

The deterministic approach, which is also pursued in this paper, copes with failures in a different way. Instead of considering a failure probability distribution for each individual component, algorithms and protocols are designed to perform well in the worst case, under some a-priori constraints on the failure behavior. [1, 2, 7, 12–14, 19, 20, 22, 23, 27]. These constraints may take the form of considering only computations with a limited overall number of faults [1, 19], limited number of faults during any single computation step [7, 13, 14, 23, 27], or during any window of first t steps [20], requiring that after some finite time there is a long enough fault-free computation [10, 15] etc. While the probabilistic model is analyzed with respect to the expected behavior, the deterministic models have been mostly analyzed for the worst case scenario.

We shall focus our attention on synchronous point-to-point distributed systems, i.e. systems in which the communication is performed by sending messages along links connecting pairs of vertices. Moreover, the vertices are synchronized by a common clock, and the delivery of every message takes exactly one time unit. This model has been widely considered [7, 8, 12–14, 20, 22, 23, 25–27] not only for its theoretical appeal, but for its practical relevance as well (e.g. many wireless networking standards, like IEEE 802.11, or GSM, operate in discrete time steps). We shall consider only one type of failure: *message loss*.

The oldest deterministic model of faults considered in this setting is the static model [1, 3], in which it is assumed that at most a fixed constant number k of messages may be lost in every step, and moreover, the failures are always located on the same links. Later, other models have been considered, too, like the dynamic model [7, 13, 14, 23, 27] in which the k failures may be located on arbitrary links in every step, linearly bounded faults [20], fractional faults [22], etc.

In [22], the fractional model has been introduced in which the number of lost messages in a given step is at most $\lfloor \alpha m \rfloor$ where m is the number of messages sent to uninformed vertices only, and $0 < \alpha < 1$ is some fixed constant. This model has, however, two major disadvantages. Considering only messages sent to uninformed vertices is rather unrealistic, since it requires a reliable acknowledgement mechanism. Furthermore, if only one message is sent in this model, it is guaranteed to be delivered, since $\lfloor \alpha \rfloor = 0$ for any $0 < \alpha < 1$.

To overcome both these undesirable features, a *fractional model with threshold* was introduced in [12]. Here, the number of messages lost in one time step is

bounded by the maximum of a fixed threshold T and a fixed fraction α of *all* sent messages. This restriction implies that if, in a given step, fewer than T messages are sent they may all be lost. On the other hand, if there are many messages sent, at least a fixed fraction $1 - \alpha$ of them is delivered. The threshold T is always assumed to be the edge connectivity minus one, since this is the largest value under which the network stays connected.

While it makes sense to modify the model of [22] without introducing the threshold (i.e. avoiding only one of the disadvantages mentioned), this has not been done so far. Hence, it is an open problem if introducing the threshold has a significant impact on the time complexity of distributed algorithms. It might be the case that using some advanced techniques one can prove that the threshold indeed does not influence the computation too much, although this would be quite surprising. We focus, however, on the analysis of the fractional model with threshold, since we consider it to be more realistic.

The fractional model with threshold avoids both problems of fractional model mentioned above, as well as undesirable properties of the static and dynamic models (where the number of faults is independent on the actual network traffic). This model has been analyzed without the restriction to greedy algorithm. In this paper, we continue in the analysis of the fractional model with threshold.

The broadcasting problem is a crucial communication task in the study of distributed systems (e.g. [21]). One vertex, called the initiator, has a piece of information that has to be distributed among all remaining vertices. Broadcasting has not only been used as a test-bed application for the study of the complexity of communication in various communication models, but has served as a building stone of many applications (e.g. [28]) as well.

We analyze the broadcasting in complete graphs and hypercubes. The broadcasting time in these graphs has been studied in the static [19], dynamic [13, 14, 23], and simple threshold [12]¹ models. In [22], broadcasting has been analyzed in the fractional model described above. Results from this paper concern the greedy offline algorithm, i.e. the algorithm that sends, in every step, messages to all uninformed neighbors of informed vertices. The results are summarized in Table 1.

We also address a natural relaxation of the broadcasting problem in which we allow a small constant number of vertices to stay uninformed in the end (a problem called *almost complete broadcasting*), and analyze the worst case time needed to solve the problem.

¹ If the number of messages sent in a given time step is less than the edge connectivity $c(G)$ in the simple threshold model, all of them may be lost. Otherwise at least one of them is delivered.

Model	K_n , chordal sense of direction	K_n unoriented	$Q_d, n = 2^d$
static	$\Theta(1)$	$\Theta(1)$	$d + 1$ [19]
dynamic	$\Theta(1)$	$\Theta(1)$ [23]	$d + 2$ [13]
fractional (greedy offline)	$\Theta(\log n)$ [22]	$\Theta(\log n)$ [22]	$O(d^3)$ [22]
simple threshold	$\Omega(n), O(n^2)$ [12]	$\Omega(n^2), O(n^3)$ [12]	$O(n^4 d^2)$ [12]

Table 1

Known time complexities of the complete broadcasting in various models.

Scenario	Almost complete broadcasting	Complete broadcasting
K_n , unoriented	$O(\log n)$	$\Omega(\log n)$ [22], $O(n^3)$ [12]
K_n , chordal sense of direction	$O(\log n)$	$\Omega(\log n)$ [22], $O(\log n)$
$K_n, \alpha < 0.55$	$O(\log n)$	$\Omega(\log n)$ [22], $O(\log n)$
Q_d	$O(d^2)$	$\Omega(d), O(n^4 d^2)$ [12]

Table 2

Results for the complete and almost complete broadcasting in the fractional model with threshold.

Our first motivation for studying this problem was to create a setting that would allow for a more practical comparison with a randomized setting, where a small probability of error is acceptable. If one considers the broadcasting problem as an optimization problem where the task is to inform as many vertices as possible, it is natural to introduce a constant additive error by allowing a constant number of vertices to stay uninformed, so that the uninformed vertices comprise at most an $O(1/n)$ fraction of all vertices and the relative error tends to zero with increasing n .

This setting also seems to be practical in many applications. One can think of sensor networks with limited battery capacities of nodes; when the battery capacity is depleted, the node goes out of operation. During the lifetime of the network, some nodes are already dead and some are still alive. When a broadcast is to be performed it might be better to “sacrifice” some active nodes in exchange for the reduction of running time which saves the energy of all other nodes.

Moreover, it seems that the study of almost complete broadcasts may help in understanding the differences among various models of faults. Indeed, for static and dynamic faults, performing the almost complete broadcast is essentially as hard as performing the complete broadcast. In the fractional model with threshold, however, it seems that the difficult part of the complete broadcast is to inform the last few vertices. The main question is whether the complexities

of the complete broadcast and almost complete broadcast differ in this setting.

For complete graphs and hypercubes, we show that the almost complete broadcast can be solved in time $O(D \log n)$, where D is the diameter of the graph and n is the number of its nodes.

Moreover, we show that if the complete graph is equipped with the chordal sense of direction, complete broadcasting can be performed in time $O(\log n)$. This is asymptotically optimal since the broadcasting time in the fractional model is a lower bound for the fractional model with threshold. Similarly we show that the broadcasting can be completed in time $O(\log n)$ for values $\alpha < 0.55$. The overview of the results can be found in Table 2. The lower bounds on complete broadcasting presented in [22] for the greedy algorithm in the fractional model can be adapted to an arbitrary algorithm in the fractional model with threshold in a straightforward way.

2 Definitions

We consider a synchronous, point-to-point distributed system with a coordinated start-up. The system consists of a number of nodes and a number of communication links connecting some pairs of nodes. The system is modeled by an undirected graph, in which vertices correspond to nodes and edges correspond to communication links. In this respect, we shall use the terms “node” and “vertex” interchangeably. Sometimes we need to argue about outgoing and incoming links; in this cases we consider a directed graph obtained from the undirected one by replacing each edge by two opposite arcs.

At the beginning of the computation all nodes are active and start performing the given protocol. The computation consists of a number of steps: at the beginning of each step, messages sent during the previous step are delivered to their destinations, then each vertex performs some local computation, possibly sending some messages², and the next step begins.

The failure model we consider is the *fractional dynamic faults with threshold* from [12], which can be described as a game between the algorithm and an adversary: in a time step t the algorithm sends m_t messages and the adversary may destroy up to

$$F(m_t) = \max\{c(G) - 1, \lfloor \alpha m_t \rfloor\}$$

of them, where $c(G)$ is the edge connectivity of the graph and α is a known, fixed constant $0 < \alpha < 1$. There is no built-in mechanism of acknowledge-

² I.e. a vertex may send different message to each of its neighbors in one step

ments, so the sender node is not informed whether a particular message was delivered or destroyed. Of course, such acknowledgements may be implemented within an algorithm, but they are treated as any other message.

We consider the problem of broadcasting, where an initiator has a piece of information to be transmitted to all remaining vertices. We call a broadcast *complete* if all vertices have the information after the termination of the algorithm. A broadcast is called *almost-complete* if there is a fixed constant c (independent on the network size) such that after the termination there are at most c uninformed vertices. Hence, to prove the existence of an almost-complete broadcasting algorithm for a family of graphs \mathcal{G} , one has to prove that there exists a constant c such that for each $G \in \mathcal{G}$ the broadcasting algorithm informs all but c vertices of G .

In all presented algorithms only the informed vertices send messages. Arcs (i.e. directed edges) leading from an informed vertex can be classified as being either active, passive or hyperactive during the computation:

Definition 1 *Let e be an arc leading from an informed vertex. We call e active if it leads to an uninformed vertex. We call an arc e passive, if some message has been delivered via the opposite arc of e . Finally, we call an arc e hyperactive if it leads to an informed vertex, and is not passive.*

If the arc e is passive, the source vertex of e is aware of the fact that the destination vertex of e has already been informed. The main idea of our algorithms is to perform appropriate number of *simple rounds* defined as follows:

Definition 2 *A simple round consists of two time steps. In the first step, every informed vertex sends a message along each of its incident arcs, excluding the passive ones.³ In the second step, all vertices that have received a message send an acknowledgement (and mark the arc as passive). Vertices that receive acknowledgement mark the corresponding arc as passive.*

For the remainder of this paper, let $0 < \alpha < 1$ be a known fixed constant, and let us denote

$$X := \frac{1}{\alpha(1 - \alpha)}$$

The rest of the paper is organized as follows. In the next two sections we present algorithms for the almost-complete broadcasting on complete graphs and hypercubes, respectively, that run in time $O(D \log n)$. Then we show how to obtain broadcast in complete graphs equipped with chordal sense of direc-

³ In this step, a message is sent via all active and hyperactive arcs. The former can inform new vertices, the latter exhibit only useless activity. However, the algorithm cannot distinguish between active and hyperactive arcs.

tion, and for unoriented complete graphs for $\alpha < 0.55$, having the same time complexity. Finally, we conclude and outline some further research directions.

3 Complete Graphs

In a complete graph K_n , all n vertices have degree $n - 1$, and $n - 1$ is also the edge connectivity. Hence, in each step t the adversary can destroy up to $\max\{n - 2, \lfloor \alpha m_t \rfloor\}$ messages, where m_t is the number of messages sent in the step t . In this section we present an algorithm that informs all but a constant number of vertices in logarithmic time. The idea of the algorithm is very straightforward – just repeat simple rounds sufficiently many times. However, the arguments given in the analysis of a simple round below hold only if there are enough informed vertices participating in the round. To satisfy this requirement two steps of a simple greedy algorithm are performed, during which each informed vertex just sends the message to all vertices. After two steps of this algorithm, the number of informed vertices is as shown in Lemma 1.

Lemma 1 *After two steps of the greedy algorithm, at least*

$$1 + \min \left\{ \frac{n}{2}, (n - 1)(1 - \alpha) \right\}$$

vertices are informed, for any $n \geq 1$.

Proof: In the first step the initiator sends $n - 1$ messages. Let $l \geq 2$ be the number of informed vertices after the first step. In the second step, $l(n - 1)$ messages are sent, and $\max\{n - 2, \alpha l(n - 1)\}$ of them are lost. We distinguish two cases:

Case 1: $\alpha l(n - 1) \leq n - 2$

In this case, at most $n - 2$ messages are lost, i.e. at least $l(n - 1) - n + 2$ are delivered. Among those delivered, at most $l(l - 1)$ could have been sent to already informed vertices. Moreover, since each uninformed vertex has at most l informed neighbors, we get that the number of informed vertices is at least

$$l + \frac{l(n - 1) - n + 2 - l(l - 1)}{l} = n - \frac{n - 2}{l}$$

Since $l \geq 2$ we get that the number of informed vertices after the two steps is at least $\frac{n}{2} + 1$.

Case 2: $\alpha l(n - 1) > n - 2$

This time, at most $\alpha l(n - 1)$ messages are lost. Using similar arguments, we

get that the number of informed vertices is at least

$$l + \frac{l(n-1)(1-\alpha) - l(l-1)}{l} = 1 + (n-1)(1-\alpha)$$

□

After these two steps, the algorithm performs a logarithmic number of simple rounds. To show that logarithmic number of simple rounds is sufficient to inform all but one vertex we first provide a lower bound on the number of acknowledgements delivered in each round, and then we show that each delivered acknowledgement decreases a certain measure function.

Theorem 1 *Let $\varepsilon > 1$ be an arbitrary constant. For large enough n it is possible to inform all but at most $X\varepsilon$ vertices in logarithmic time. Moreover, the number of remaining hyperactive arcs is at most $X(n-2)$.*

Proof: At the beginning, two steps of the greedy algorithm are executed. Then, a logarithmic number of simple rounds is performed. Now consider the situation at the beginning of the i -th round. Let k_i be the number of uninformed vertices, and h_i the number of hyperactive arcs. We claim that if $k_i > X\varepsilon$ or $h_i > X(n-2)$ then at least

$$[k_i(n - k_i) + h_i] (1 - \alpha)^2$$

acknowledgements are delivered in this round. Since there are $k_i(n - k_i) + h_i$ messages sent in this round, in order to prove the claim it is sufficient to show that

$$\alpha(1 - \alpha) [k_i(n - k_i) + h_i] \geq n - 2.$$

Indeed, in this case, it holds that

$$\alpha [k_i(n - k_i) + h_i] \geq n - 2 = c(G) - 1,$$

hence at most

$$\alpha [k_i(n - k_i) + h_i]$$

messages are lost. This means that at least

$$(1 - \alpha) [k_i(n - k_i) + h_i]$$

messages are delivered, and the same number of acknowledgements are sent. Due to the assumption, at most

$$\alpha(1 - \alpha) [k_i(n - k_i) + h_i]$$

acknowledgements are lost, so at least

$$[k_i(n - k_i) + h_i] (1 - \alpha)^2$$

of them are delivered.

Obviously, if $h_i > X(n-2)$ the inequality

$$\alpha(1-\alpha)[k_i(n-k_i) + h_i] \geq n-2$$

holds, so consider the case $k_i > X\varepsilon$. We prove that in this case

$$k_i(n-k_i) \geq X(n-2),$$

i.e.

$$k_i^2 - nk_i + X(n-2) \leq 0.$$

Let

$$f(n) := 1/2 \left(n - \sqrt{n^2 - 4X(n-2)} \right).$$

The roots⁴ of the equation $k_i^2 - nk_i + X(n-2) = 0$ are $f(n)$ and $n - f(n)$, so we want to show that $f(n) \leq k_i \leq n - f(n)$. We have

$$\begin{aligned} \lim_{n \rightarrow \infty} f(n) &= \lim_{n \rightarrow \infty} 1/2(n - \sqrt{n^2 - 4X(n-2)}) = \\ \lim_{n \rightarrow \infty} 1/2(n - \sqrt{n^2 - 4X(n-2)}) \frac{n + \sqrt{n^2 - 4X(n-2)}}{n + \sqrt{n^2 - 4X(n-2)}} &= \\ \lim_{n \rightarrow \infty} 1/2 \frac{n^2 - n^2 + 4X(n-2)}{n + \sqrt{n^2 - 4X(n-2)}} &= \\ \lim_{n \rightarrow \infty} 1/2 \frac{4X(1 - 2/n)}{1 + \sqrt{1 - 4X(1/n - 2/n^2)}} &= X \end{aligned}$$

so we get that $k_i > X\varepsilon > f(n)$ holds for large enough n . Hence, the only remaining step is to show the inequality $k_i \leq n - f(n)$. From Lemma 1 it follows that

$$n - k_i > \min \{n/2, (n-1)(1-\alpha)\}.$$

Since $f(n) < n/2$, if $n - k_i > n/2$ it holds $k_i < n - f(n)$. So let us suppose that $n - k_i > (n-1)(1-\alpha)$, i.e. $k_i < 1 + \alpha(n-1)$. Let

$$n \geq \frac{\varepsilon + \alpha(1-\alpha)^2}{\alpha(1-\alpha)^2}.$$

Then it holds for large enough n that

$$k_i < 1 + \alpha n - \alpha \leq n - \frac{\varepsilon}{\alpha(1-\alpha)} = n - \varepsilon X \leq n - f(n).$$

⁴ Assume that n is large enough such that $f(n)$ is real number.

We have proved that if $k_i > X\varepsilon$ or $h_i > X(n-2)$ then at least

$$[k_i(n - k_i) + h_i] (1 - \alpha)^2$$

acknowledgements are delivered in round i .

To conclude the proof we show that after logarithmic number of iterations we get $k_i \leq X\varepsilon$ and $h_i \leq X(n-2)$. Let $M_i := 2(n-1)k_i + h_i$; then every delivered acknowledgement decreases M_i by at least one: indeed, if the acknowledgement was delivered over a hyperactive arc, h_i decreases by 1. If, on the other hand, the acknowledgement was delivered over an active arc, the number of uninformed vertices is decreased by at least one, and the number of hyperactive arcs is increased by at most $2n-3$ (new hyperactive arcs are between the newly informed vertex and any other vertex, with the exception of the arc that delivered the acknowledgement which is passive).

From Lemma 1 it follows that either $n - k_i > n/2$ or $n - k_i > (n-1)(1-\alpha)$. In the first case it follows that at least

$$(1 - \alpha)^2 [k_i(n - k_i) + h_i] > (1 - \alpha)^2 [k_i n/2 + h_i] \geq \frac{(1 - \alpha)^2}{4} M_i$$

acknowledgements are delivered. In the second case we get that at least

$$(1 - \alpha)^2 [k_i(n - k_i) + h_i] > (1 - \alpha)^2 [k_i(n-1)(1-\alpha) + h_i] \geq \frac{(1 - \alpha)^3}{2} M_i$$

acknowledgements are delivered. Let

$$c := \min\left\{\frac{(1 - \alpha)^2}{4}, \frac{(1 - \alpha)^3}{2}\right\},$$

then obviously every iteration decreases the value of M_i at least by factor c . Since the value of M at the beginning of the algorithm is $M_1 = O(n^2)$, $\log_{1/c} M_1 = O(\log n)$ steps are sufficient to inform all but a constant number (at most $X\varepsilon$) of vertices and to ensure that the number of remaining hyperactive arcs is linear (at most $X(n-2)$). \square

4 Hypercubes

In this section we consider d -dimensional hypercubes. The hypercube Q_d has 2^d vertices, and both diameter and edge connectivity are d . We present an algorithm that informs all but a constant number of vertices in time $O(d^2)$.

The general idea is the same as for complete graphs: first we perform two initialization steps to make sure there are enough informed vertices for the

subsequent analysis to hold. Next, simple rounds are repeated for a sufficient number of times. The analysis, however, is more complicated in this case.

The next lemma covers the initialization steps. In the first step, the initiator sends a message to all its neighbors, and at least one of these messages is delivered. In the second step, the initiator sends a message to all its neighbors again; moreover, each of the vertices informed in the first step sends a message to all its neighbors except the initiator.

Lemma 2 *After the first two steps of the algorithm, at least $\frac{1-\alpha}{2}(2d-1)$ vertices are informed.*

Proof: In the first step, the initiator sends d messages. Since at most $d-1$ can be lost, some $l > 0$ of them are delivered. In the second step, the initiator sends again d messages, but at the same time, each of the informed vertices sends $d-1$ messages to all its neighbors except initiator. Hence, $d+l(d-1)$ messages are sent in the second step. Let us distinguish two cases:

If $d-1$ messages are lost, then $d+(l-1)(d-1)$ messages are delivered. l messages from the initiator can be delivered to the already informed vertices which leaves $d+(l-1)(d-1)-l$ messages that enter uninformed vertices. Since at most l messages can be destined to the same vertex, The number of informed vertices after two steps is at least $1+l+\frac{d+(l-1)(d-1)-l}{l} \geq (1/2)(2d-1)$.

If at most $\alpha[d+l(d-1)]$ messages are lost, then at least $(1-\alpha)[d+l(d-1)]-l$ messages arrive into uninformed vertices. Hence, there are at least $\frac{1-\alpha}{l}[d+l(d-1)]+l \geq \frac{1-\alpha}{2}(2d-1)$ informed vertices. \square

For the rest of this section we suppose that there are at least $\frac{1-\alpha}{2}(2d-1)$ informed vertices. We show that after $O(d^2)$ simple rounds all but some constant number of vertices are informed, and there are only linearly many hyperactive arcs. At the end of this section, we shall be able to prove the following theorem.

Theorem 2 *Let $\varepsilon > 1$ be an arbitrary constant. For large enough d it is possible to inform all but at most $X\varepsilon$ vertices of Q_d within $O(d^2)$ time steps. Moreover, the number of remaining hyperactive arcs is at most $X(d-1)$.*

In our analysis we need to assert that enough acknowledgements are delivered, given the number of informed vertices. To bound the number of sent messages, we rely heavily upon the following isoperimetric inequality due to Chung et. al. [9]:

Claim 1 [9] *Let S be a subset of vertices of Q_d . The size of the edge boundary of S , denoted as $\partial(S)$ is defined as the number of edges connecting S to $V(Q_d) \setminus S$. Let $\partial(k) = \min_{|S|=k} \partial(S)$, and let \lg denote the logarithm of base 2. It holds*

that

$$\partial(k) \geq k(d - \lg k)$$

The first step in the analysis is to prove that if there are enough uninformed vertices, or enough hyperactive arcs at the beginning of a round i , then sufficiently many acknowledgements are delivered in this round:

Lemma 3 *Consider a d -dimensional hypercube with k non-informed vertices and h hyperactive arcs. Let $\varepsilon > 1$ be an arbitrary constant, and let $k > X\varepsilon$ or $h > X(d-1)$. Then in the second step of a simple round at least $(1-\alpha)^2(h + \partial(k))$ acknowledgements are delivered.*

Proof: Let S be the set of informed vertices. In the first step of the round, $h + \partial(S)$ messages are sent. Since the edge boundary of informed and uninformed vertices is the same, at least $h + \partial(k)$ messages are sent in the first step of the round. We prove that $\alpha(h + \partial(k)) \geq d - 1$, so in the first step at most $\alpha(h + \partial(k))$ messages are lost, and at least $(1 - \alpha)(h + \partial(k))$ of them are delivered. Next we prove that $\alpha(1 - \alpha)(h + \partial(k)) \geq d - 1$, so in the second step at least $(1 - \alpha)^2(h + \partial(k))$ messages are delivered. Since $1 - \alpha < 1$, it is sufficient to prove that $\alpha(1 - \alpha)(h + \partial(k)) \geq d - 1$.

If $h > X(d-1)$ then obviously $h + \partial(k) \geq X(d-1)$ and the statement holds. Next, let us consider the case when $h > X/\varepsilon$. We distinguish three cases and prove that in each case $\partial(k) \geq X(d-1)$. Let $\delta := 1 - 1/\varepsilon$, i.e. $1/(1 - \delta) = \varepsilon$.

Case 1: $k \leq 2^{\delta d}$

In this case it holds that $\partial(k) \geq k(d - \lg k) \geq kd(1 - \delta)$. Since $k > X/(1 - \delta) = X\varepsilon$, we get $\partial(k) \geq Xd$.

Case 2: $2^{\delta d} \leq k \leq 2^d \left(1 - \frac{1}{e}\right)$, where e is the base of natural logarithm

In this case $\partial(k) \geq k(d - \lg k) \geq 2^{\delta d} \left(d - d - \lg \left(1 - \frac{1}{e}\right)\right) = 2^{\delta d} \lg \frac{e}{e-1} \geq 0.6 \cdot 2^{\delta d}$. Since X is constant, for large enough d it holds that $\partial(k) \geq 0.6 \cdot 2^{\delta d} > X(d-1)$.

Case 3: $2^d \left(1 - \frac{1}{e}\right) \leq k$

First, let us consider a function $f(x) := x(d - \lg x)$, for $x \in \langle 0, 2^d \rangle$. Since $f'(x) = d - 1/\ln 2 - \lg x$, $f(x)$ is increasing for $x \in \langle 0, 2^d/e \rangle$ and decreasing for $x \in \langle 2^d/e, 2^d \rangle$.

Obviously, the edge boundary of uninformed vertices $\partial(k)$ is the same as the edge boundary of informed vertices $\partial(2^d - k)$. Hence, we get $\partial(k) \geq f(2^d - k)$. Since $2^d - k \leq 2^d \frac{1}{e}$, the minimum of $f(2^d - k)$ is attained for the minimal value of $2^d - k$. From Lemma 2 we know that $2^d - k > \frac{1-\alpha}{2}(2d-1)$, so $\partial(k) \geq$

$f\left(\frac{1-\alpha}{2}(2d-1)\right) = \frac{1-\alpha}{2}(2d-1)\left(d - \lg \frac{1-\alpha}{2}(2d-1)\right) = (1-\alpha)d^2 - O(d \lg d)$.
Hence, for large enough d we get $\partial(k) \geq X(d-1)$. \square

In the rest of the proof of Theorem 2 we show that $O(d^2)$ simple rounds are sufficient to inform almost all vertices. The analysis is divided into two parts. In the first part we prove that within $O(d^2)$ rounds at least $2^d/3$ vertices are informed. In the second part we show that another $O(d^2)$ rounds are sufficient to finish the algorithm. Furthermore, we need the following lemma⁵ for our analysis.

Lemma 4 *Let $x \geq 2$. It holds that $\lg \frac{x+1}{x} \geq \frac{1}{x}$.*

Proof: The statement is equivalent to:

$$\forall x \geq 2 : \frac{1}{x} \geq 2^{\frac{1}{x}} - 1$$

Substituting $y := \frac{1}{x}$:

$$\forall y \in (0, 1/2) : y \geq 2^y - 1$$

For $y = 0$ the equality holds. Hence it is sufficient to prove that the derivative of the left side is larger than the derivative of the right side for $y \in (0, 1/2)$, i.e. $1 \geq 2^y \ln 2$, which obviously holds. \square

Lemma 5 *After performing $O(d^2)$ simple rounds on Q_d at least $2^d/3$ vertices are informed.*

Proof: Let $l := 2^d - k$ be the number of informed vertices and b be the number of passive arcs at the beginning of some simple round. Obviously $b \leq ld$. Since we can assume that $l < 2^d/3$, the conditions of Lemma 3 are met, and at least $\beta\partial(k)$ acknowledgements are delivered in one simple round, where $\beta := (1 - \alpha)^2$. Furthermore, the edge boundary of informed vertices is also the boundary of uninformed vertices, so the number of delivered acknowledgements is at least $\beta\partial(l)$. Because every delivered acknowledgement adds one passive arc, the number of passive arcs grows at least to $b' = b + \beta\partial(l)$ after this round.

As in Lemma 3, let us consider a function $f(x) := x(d - \lg x)$, for $x \in \langle 0, 2^d \rangle$. Since $f'(x) = d - 1/\ln 2 - \lg x$, $f(x)$ is increasing for $x \in \langle 0, 2^d/e \rangle$ and decreasing for $x \in \langle 2^d/e, 2^d \rangle$.

As $b/d \leq l \leq 2^d/3 \leq 2^d/e$ it holds that $\partial(b/d) \leq \partial(l)$. Hence we have the following lower bound on b' :

$$b' \geq b + \beta\partial\left(\frac{b}{d}\right) \geq b + \beta\frac{b}{d}\left(d - \lg \frac{b}{d}\right) \geq b\left(1 + \beta\frac{d - \lg \frac{b}{d}}{d}\right)$$

⁵ Which is in fact a consequence of a known inequality $\ln(x+1) \geq \frac{x}{x+1}$

The lower bound on b implies the inequality $\lg \frac{b}{d} \leq d + \lg(1/3)$. Hence it holds

$$b' \geq b \left(1 + \beta \frac{-\lg(1/3)}{d} \right) = b \left(1 + \frac{1}{\frac{d}{\beta \lg 3}} \right)$$

We have shown that the number of passive arcs grows exponentially with number of simple rounds performed. As it cannot grow above $d2^d/3$ without informing at least $2^d/3$ vertices, we can estimate an upper bound on number of required simple rounds:

$$T \leq \frac{\lg(d2^d/3)}{\lg \left(1 + \frac{1}{\frac{d}{\beta \lg 3}} \right)}$$

For large enough d , Lemma 4 is applicable, hence proving the Lemma:

$$T \leq \lg(d2^d/3) \frac{d}{\beta \lg 3} = O(d^2)$$

□

Lemma 6 *Let $\varepsilon > 1$ be an arbitrary constant, and let $k_i \leq (2/3)2^d$ be the number of uninformed vertices and h_i the number of hyperactive arcs of Q_d at the beginning of round i . Then after $O(d^2)$ simple rounds there are at most $X\varepsilon$ uninformed vertices and at most $X(d-1)$ hyperactive arcs.*

Proof: Similarly to the proof of Theorem 1 let us consider the measure $M_i := 2dk_i + h_i$. The requirements of the Lemma ensure that $M_i \leq O(d2^d)$. It is easy to see that M_i decreases with every acknowledgement delivered: if the acknowledgement is delivered over a hyperactive arc, the value of h_i decreases by 1. If it is delivered over an active arc, new vertex is informed, hence the value of k_i decreases by 1 and the value of h_i increases by at most $2d-1$.

We show that the value of M_i decreases by a certain multiplicative factor in every simple round as long as the requirements of Lemma 3 hold. In one simple round at least $\beta(h_i + \partial(k_i))$ acknowledgements are delivered, where $\beta := (1-\alpha)^2$. Hence, the value M_i decreases to at most:

$$\begin{aligned} M_{i+1} &\leq 2dk_i + h_i - \beta(h_i + \partial(k_i)) \leq h_i(1-\beta) + 2dk_i - \beta k_i(d - \lg k_i) = \\ &= h_i(1-\beta) + 2dk_i \left(1 - \beta + \beta \frac{\lg k_i}{d} \right) \end{aligned}$$

Using the inequality $\lg k_i \leq d + \lg(2/3)$ yields:

$$M_{i+1} \leq h_i(1-\beta) + 2dk_i \left(1 + \frac{\beta \lg(2/3)}{d} \right)$$

Hence for large enough d it holds:

$$M_{i+1} \leq (h_i + 2dk_i) \left(1 + \frac{\beta \lg(2/3)}{d} \right)$$

Since the requirements of the Lemma ensures that $M_i \leq (7/3)d2^d$, the requirements of Lemma 3 can hold for at most

$$T := \frac{\lg\left(\frac{7}{3}d2^d\right)}{\lg\left(1 + \frac{1}{\beta \lg(2/3)}\right)}$$

time steps. According to Lemma 4 for large d it holds that

$$T \leq \lg\left(\frac{7}{3}d2^d\right) \frac{d}{\beta \lg(2/3)} = O(d^2)$$

which concludes the proof. \square

Combining Lemma 2 with Lemma 5 and Lemma 6 completes the proof of Theorem 2.

5 Complete Broadcast in Complete Graphs

In Section 3 we have shown how to inform all but some constant number of vertices in a complete graph K_n in time $O(\log n)$. A natural question is to ask if it is possible to also inform the remaining vertices in the same time complexity. In this section we partially answer this question. In particular, we show in the following subsection that if the graph is equipped with a chordal sense of direction, then the complete broadcasting can be performed in time $O(\log n)$. In the subsequent subsection, we show that if the constant $\alpha < 0.55$, complete broadcast can be performed in time $O(\log n)$ without the sense of direction, too.

5.1 Chordal Sense of Direction

Let us consider a complete graph with a fixed Hamiltonian cycle \mathcal{C} (unknown to the vertices). We say that the complete graph has a chordal sense of direction if in every vertex the incident arcs are labeled by the clockwise distance on \mathcal{C} (see Figure 1). The notion of a sense of direction has been defined formally for general graphs, and it has been known to significantly reduce the complexity of many distributed tasks (e.g. [17, 18]).

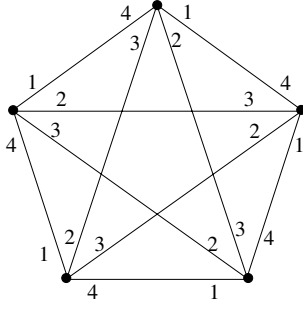


Fig. 1. K_5 with the chordal sense of direction.

We show how to perform a complete broadcast on a complete graph with the sense of direction in time $O(\log n)$. The process consists of three steps. First, using Theorem 1, all but a constant number of vertices are informed. In the second phase the information is delivered to all but one vertex. In the last phase the remaining single vertex is informed.

The sense of direction is essential to our algorithm. Since there is a unique initiator of the broadcasting, all vertices can derive unique identifiers defined as their distance on \mathcal{C} from the initiator. Furthermore, the sense of direction allows each vertex to know the identifier of a destination vertex of any of its incident arcs.

Lemma 7 *It is possible to inform all vertices but one on complete graphs with chordal sense of direction in time $O(\log n)$. Furthermore, after finishing the algorithm vertex 0 or vertex 1 knows a constant number of candidates for the uninformed vertex.*

Proof: The outline of the algorithm is as follows: at first the algorithm from Theorem 1 is performed, which ensures that all but a constant number of vertices are informed. Afterwards a significant group of vertices negotiates a common set U of *candidates* for uninformed vertices, such that all uninformed vertices are in U and the size of U is constant. The vertices then cooperate to inform all vertices in U but one. As a side effect, the set U will be known to vertex 0 or vertex 1, hence satisfying the second claim of the lemma. Now we present this algorithm in more detail:

Phase 1 Run the algorithm from Theorem 1. This phase takes $O(\log n)$ time and ensures that there are at most $X\varepsilon$ uninformed vertices and at most $X(n-2)$ hyperactive arcs.

Phase 2 Each vertex v that has at most $3X(1+\varepsilon)$ non-passive (i.e. active or hyperactive) links leading to the set of vertices U_v sends a message containing U_v to vertices with number 0 and 1.

Now we show that at least one of these messages is delivered. It is easy to see that there are at least $2n/3$ vertices satisfying the above-mentioned condition, otherwise there would be more than $n/3$ vertices with at least

$3X(1 + \varepsilon)$ non-passive links, so there would be more than $nX(1 + \varepsilon)$ active or hyperactive arcs. But since the number of uninformed vertices is at most $k \leq X\varepsilon \leq n/2$ for large n , there are $k(n - k) \leq X\varepsilon(n - X\varepsilon)$ active arcs. So the total number of active or hyperactive arcs is at most $X\varepsilon(n - X\varepsilon) + X(n - 2) \leq Xn(1 + \varepsilon)$, which is a contradiction.

The rest of the algorithm will be time-multiplexed into two parts. In even time steps, the case that the vertex 0 received a message in phase 2 is processed. In odd time steps, the case that the vertex 1 received a message is processed analogously. Hence, we can restrict to the first case in the rest of the algorithm description. As there are only two cases the asymptotic complexity of the algorithm is unaffected by the multiplexing.

Phase 3 The vertex 0 received at least one message containing a set of possibly uninformed vertices. It is obvious that the set of uninformed vertices is a subset of every received message. Hence the set U can be defined as the intersection of the received messages: Indeed, every uninformed vertex is in U and the size of U is at most $3X(1 + \varepsilon) = O(1)$. The set U is then distributed using the algorithm in Theorem 1 among at least $n - X\varepsilon$ vertices in time $O(\log n)$.

Phase 4 There are at least $n - X\varepsilon$ vertices aware of the set U . In this phase they cooperate to inform all but one vertex in U , using an idea similar to Lemma 2 in [12]: every vertex aware of the set U iterates through all pairs $[i, j]$ ($i, j \in U$) in lexicographical order; in each time step it sends the original message to both vertices i and j . Since in each time step at least $2n - X\varepsilon$ messages are sent, at least one of them is delivered (for large enough n). As all vertices process the same pair $[i, j]$ in every time step, this ensures that a new vertex is informed whenever both i and j were uninformed. Hence, at the end of this phase all vertices but one are informed. The time complexity of this phase is $O(|U|^2) = O(1)$.

It is obvious that after finishing the Phase 4 the claim of the Lemma holds. □

Finally, we show how to inform the last remaining vertex, thus proving the following theorem:

Theorem 3 *It is possible to perform broadcasting on complete graphs with chordal sense of direction in time $O(\log n)$.*

Proof: We present an algorithm for solving the broadcasting problem:

Phase 1 The algorithm from Lemma 7 is used. This takes $O(\log n)$ time, all vertices but one are informed and the vertex 0 or the vertex 1 knows a set U of constant size containing candidates for the uninformed vertex.

The rest of the algorithm is multiplexed into two parts, treating these two cases separately. In the remaining of the description we assume that

the vertex 0 knows the set U .

Phase 2 The algorithm from Lemma 7 is used to broadcast the set U , together with the original information, to all vertices but one. This takes $O(\log n)$ time again.

After the Phase 2 is finished, two cases are possible: either the uninformed vertex of the Phase 2 is different from or is the same as the uninformed vertex of the Phase 1. In the former case all vertices are informed. The rest of the algorithm handles the latter case.

Phase 3 If not all vertices are informed, then there is a single uninformed vertex v . Furthermore, every informed vertex knows the set U of constant size such that $v \in U$. Every informed vertex iterates through the set of U ; in i -th time step of the current phase it sends the message to i -th member of U . Eventually, the uninformed vertex is processed. Since all $n - 1$ informed vertices are doing the same, exactly $n - 1$ messages are sent to the uninformed vertex, hence finishing the broadcast.

The time complexity of the Phase 1 and Phase 2 is $O(\log n)$; the time complexity of the Phase 3 is $O(|U|) = O(1)$. Hence the algorithm correctly solves the broadcasting on complete graphs in time $O(\log n)$. \square

5.2 Without Sense of Direction

As a last result in this paper we show that it is possible to perform broadcasting on complete graphs in time $O(\log n)$ for small values of α (i.e. $\alpha \lesssim 0.55$) even without the sense of direction. The idea is to use the algorithm from Theorem 1 to inform all but constantly many vertices. Next, instead of repeating 2-step simple rounds, some $\log n$ -step extended rounds are repeated, such that each extended round informs a yet uninformed vertex. During an extended round messages are sent for $O(\log n)$ steps in such a way that in every step the number of hyperactive arcs is decreased by some factor⁶ unless a new vertex is informed.

Theorem 4 *Let $\alpha \lesssim 0.55$. Then it is possible to perform broadcasting on complete graphs without sense of direction in time $O(\log n)$.*

Proof: The algorithm is described as Algorithm 1.

At first, the algorithm from Theorem 1 is performed, ensuring that there are at most $k \leq X\varepsilon$ uninformed vertices and at most $h \leq X(n - 2)$ hyperactive arcs (X and ε have the same meaning as in Theorem 1). The purpose of one iteration of the loop on lines 3–14 is to inform at least one uninformed vertex. Taking $L_1 := X\varepsilon = O(1)$ ensures that all vertices will be informed.

⁶ In this part we need the assumption that α is small enough

Algorithm 1 Complete graphs without sense of direction.

```
1: perform almost-complete broadcast according to Theorem 1
2: let  $k$  denote the number of uninformed vertices, let  $h$  denote the number
   of hyperactive arcs
3: loop  $L_1$  times // Perform  $L_1$  extended rounds
4:   loop  $L_2(n)$  times // In each iteration  $h$  decreases by a constant
   factor
5:      $E :=$  set of all currently active or hyperactive arcs;  $P := \emptyset$ 
6:     loop  $L_3$  times
7:       send the message via all arcs in  $E \cup P$ 
8:        $P := P \cup \left\{ e \mid \begin{array}{l} \text{a message has been delivered in this step} \\ \text{via the opposite arc of } e \end{array} \right\}$ 
9:     end loop
10:  end loop
11:  loop  $L_4(n)$  times // Inform new vertex and decrease  $a$ 
12:    perform one simple round
13:  end loop
14: end loop
```

The values of L_1 , $L_2(n)$, L_3 and $L_4(n)$ are specified in the analysis of the algorithm, such that $L_1, L_3 = O(1)$ and $L_2(n), L_4(n) = O(\log n)$.

The loop on lines 4–10 reduces the number of hyperactive arcs to zero unless a new vertex is informed. One iteration of this loop either informs a new vertex or reduces the number of hyperactive arcs from h to $(1 - Y/2)h$, where $0 < Y < 1$ is a constant (depending on α) defined later. Hence the number of hyperactive arcs decreases exponentially with number of iterations of the loop and $\log_{1/(1-Y/2)} h$ iterations are sufficient to eliminate all hyperactive arcs. Since the condition $h \leq X(n - 2)$ holds before every execution of the loop (this is provided either directly by Theorem 1 or by the loop on lines 11–13), we can define $L_2 := \log_{1/(1-Y/2)}(X(n - 2)) = O(\log n)$.

Now we describe one iteration of the loop on lines 4–10. We distinguish two types of arcs that are hyperactive at the beginning of the considered iteration: an arc e is a *single hyperactive arc* if and only if it is hyperactive and the opposite arc of e is passive at the beginning of the iteration. Otherwise (i.e. if both e and the opposite arc of e are hyperactive at the beginning of the iteration), e is a *double hyperactive arc*.

Let E be the set of all active or hyperactive arcs at the beginning of the iteration, and P be the set of all arcs opposite to arcs through which some message has been delivered in the current iteration. Furthermore, let k' be the number of uninformed vertices at the beginning of the current iteration, h' be the number of hyperactive arcs at the beginning of the current iteration and $p = |P \setminus E|$ be number of arcs in P that were passive at the beginning

of the current iteration. It clearly holds that $|E| = k'(n - k') + h'$ and that $k'(n - k') + h' + p$ messages are sent on every execution of line 7. Since at least $n - 1$ messages are lost (because we may assume that no new vertex is informed), at most $\alpha(k'(n - k') + h' + p)$ of them are lost, i.e. at least $(1 - \alpha)(k'(n - k') + h' + p)$ are delivered.

Now assume by contradiction that the number of hyperactive arcs does not decrease below $(1 - Y/2)h'$, and no new vertices are informed during the current iteration of the loop on lines 4–10. Consider any message delivered over an arc e which is a double hyperactive arc or an arc in $P \setminus E$; it is easy to see that the opposite arc of e is passive after the delivery and that it was hyperactive at the beginning of the iteration. This fact yields that at most $(Y/2)h'$ messages are delivered over a double hyperactive arc or an arc in $P \setminus E$ on any execution of line 7.

Now we show a lower bound on the number of messages that pass over double hyperactive arcs or arcs in $P \setminus E$ or single hyperactive arcs whose opposite arcs are not in $P \setminus E$. Intuitively, every such message ensures some progress of the algorithm, since either an arc is made passive (in the first two cases) or a new arc is added to $P \setminus E$ (in the third case). As no messages passes over active arcs by our assumption, and at most p messages pass over single hyperactive arcs whose opposite arcs are in $P \setminus E$, there are at least $(1 - \alpha)(k'(n - k') + h' + p) - p$ messages satisfying one of these three cases. Using the inequalities $k'(n - k') \geq n - 1$ and $p \leq h'$ yields $(1 - \alpha)(k'(n - k') + h' + p) - p \geq (1 - \alpha)(n - 2) + (1 - 2\alpha)h'$. Because $h' \leq X(n - 2)$ which is equivalent to $(n - 2) \geq \alpha(1 - \alpha)h'$, we have $(1 - \alpha)(k'(n - k') + h' + p) - p \geq (1 - \alpha - 2\alpha^2 + \alpha^3)h'$. Defining $Y := 1 - \alpha - 2\alpha^2 + \alpha^3$, which is positive and less than one by the assumption of the Lemma, we have shown that there are at least Yh' messages satisfying one of the three cases.

However, at most $(Y/2)h'$ of them satisfies the first two cases, hence there are at least $(Y/2)h'$ arcs added to P in every execution of line 8. So taking $L_3 := 2/Y + 1$ ensures that P contains opposite arcs to all single hyperactive arcs at the beginning of the last iteration of the loop on lines 6–9. However, this is a contradiction with the fact that new arcs are added to P at line 8.

We conclude the proof with the analysis of the loop on lines 11–13. In the first iteration of the loop a new vertex is informed, because there are no hyperactive arcs left after the loop on lines 4–10 finished (unless the new vertex has already been informed in that loop). Due to Theorem 1, next $O(\log n)$ iterations are sufficient to ensure that $h \leq X(n - 2)$, which is an invariant required by the loop on lines 4–10. Hence putting $L_4(n) := O(\log n)$ (according to Theorem 1) is sufficient to make the algorithm work correctly in time $L_1(L_2(n)L_3 + L_4(n)) = O(\log n)$. \square

6 Conclusions, Open Problems, and Further Research

We have studied the problem of almost complete broadcast under the model of fractional dynamic faults with threshold. We showed that both in complete graphs and in hypercubes, it is possible to inform all but constantly many vertices in time $O(D \log n)$ where D is the diameter of the graph and n is the number of vertices.

Moreover, we have proved that if the complete graph is equipped with the chordal sense of direction, or the parameter $\alpha < 0.55$, a complete broadcast can be performed in time $O(\log n)$.

This research leaves many open questions and directions for further research, from which we mention at least a few. One obvious question is to ask if it is possible to perform a complete broadcast in complete graphs also for large values of α in polylogarithmic time. The difficulty of broadcasting in the fractional dynamic model with threshold stems from the fact that, in order to inform the last few vertices, all informed vertices must cooperate very tightly. In general, the relationship between the almost complete and complete broadcast in various models is worth studying. We have also not considered non-constant values of α . It would be interesting to extend our results to more general classes of graphs.

We finish by noting that there is a lack of any non-trivial lower bounds in the model of fractional faults with threshold.

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