Broadcasting with Many Faulty Links*

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Abstract

We study the problem of broadcasting in point-to-point networks with faulty links. The usual approach in which the number of faulty links in each step is bounded by a fixed constant does not reflect the intuition that the probability of a certain link to fail while transmitting a particular message is a fixed constant. In our model the number of faulty links in a given step depends on the total number of links used in that step. We are interested in fault-resilience of networks for broadcast in this model: in which class of networks the broadcasting time is proportional to their diameter. We show that toroidal grids of constant dimension are fault-resilient in our model, but on the other hand complete d-ary trees and cliques are not.

Keywords

Distributed Computing, Broadcast, Fault Tolerance, Tori

1 Introduction

We investigate the properties of information dissemination in a network with faulty links. Our goal is to find relation between structural properties of a network and the extent to which the broadcasting of a piece of information may be slowed down by the presence of faults.

The information flow in a point-to-point communication network is traditionally modeled by the broadcast problem (e.g. [10, 11]): one node of the network has a message which should be transmitted to all other nodes. This kind of network communication has numerous applications in fault-tolerant parallel and distributed computing.

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In our research we have adopted the well known synchronous “shouting” (or “all port”) model in which the original message is delivered in a series of synchronous steps such that in each step every informed node sends the message to all its uninformed neighbors. Clearly, without the presence of faults this process delivers the message to every node after $diam_G$ steps, where $diam_G$ denotes the diameter of the graph.

One approach to modeling the impact of faults is to study models in which certain number of links is faulty, i.e. they do not deliver any information. An important fact is that in different time steps the status of a given link may change between *faulty* and *operational*, but the status of each link is fixed within one step.

A well studied model of dynamic faults (e.g. [2, 6, 7, 8, 9]), introduced by Santoro and Widmayer [13], allows a fixed number of k links to be faulty during any given step. In this model the broadcasting may be successfully accomplished only if the edge-connectivity of the underlying network exceeds k. The main question was to determine the class of networks $\{G_n\}$ that are *fault-resilient* for broadcast, i.e. there is a constant $c > 0$ such that the broadcasting time on a graph G_n is upper bounded by $c \cdot diam_{G_n}$ for all n. In [6, 8, 9] it has been proven that certain interconnection topologies (including tori, hypercubes and star graphs) are fault-resilient for broadcast in this model (i.e. broadcasting is completed in $O(diam_G)$ time in the presence of $\Delta - 1$ dynamic faults, where Δ is the edge-connectivity of G). Moreover, in [2] a class of networks is presented which are highly non-fault-resilient for broadcast in this model.

In an effort to make the model less restrictive and to capture the intuition that the probability of a certain link to fail while transmitting a particular message is a fixed constant, we relax the model by allowing a non-constant number of faulty links in the following way: consider a particular step in the broadcast where each informed vertex is trying to send the information to all its uninformed neighbors. We call the links that are going to be used in this process “active” and from among these we allow a fixed fraction to fail.

An alternative way of relaxing the model was studied in [3, 4, 5], where the number of faulty transmissions during the first i time steps was bounded by αi for a fixed constant α and all natural i.

The main question we ask is: *“In which graphs the faults cannot asymptotically slow down the broadcasting?”*. We show that in cliques K_n the broadcasting time is $\Theta(\log n)$ and in complete binary trees of height n the broadcasting time is at least $\Omega\left(2^{\frac{n}{2}}\right)$. The latter result generalizes readily to complete d-ary trees.

On the other hand, we show that for n-dimensional grids with constant dimension the broadcasting time remains $O(diam_G)$ regardless of the presence of faults.

The rest of this paper is organized as follows: in Section 2 we introduce our model more formally, in Section 3 we study the broadcasting time in various topologies and conclude in Section 4.

2 The Model

Let us define our model of faulty broadcast in a more formal way. As usual, the network will be modeled by a graph $G = (V, E)$ in a straightforward way. We first specify what we mean by a broadcast:

Definition 1 *Consider a graph $G = (V, E)$ and a vertex $w \in V$. A* broadcast *(starting from w) is a sequence $\{w\} = V_0 \subseteq V_1 \subseteq \cdots \subseteq V_t = V$ of subsets of V. The number t is called* broadcasting time *and the set V_i represents the vertices which are informed after i steps.*

In general, it may not be the case that a vertex always tries to inform *all* its uninformed neighbors, so we will think in terms of a *broadcasting algorithm*. A broadcasting algorithm $\mathcal{A}$ constructs a broadcast in an incremental way: given a set V_i of informed vertices, $\mathcal{A}$ selects a set of *active edges* E_i^*. Now it is the time for the *adversary* to select a set F_i of *faulty edges* at step i. The remaining active edges E_i are then used to deliver the information. Hence in each step of the broadcasting, the following takes place:

1. $\mathcal{A}$ selects active edges $E_i^* \subseteq \partial V_i$, where ∂S denotes edge boundary of a set $S \subseteq V$, i.e. $\partial S = \{e \in E \mid e = (u, v),\ u \in S,\ v \in V - S\}$.
2. adversary selects faulty edges $F_i \subseteq E_i^*$
3. $V_{i+1} := V_i \cup \{v \in V \mid \exists e \in E_i,\ e = (u, v),\ u \in V_i\}$, where $E_i = E_i^* - F_i$.

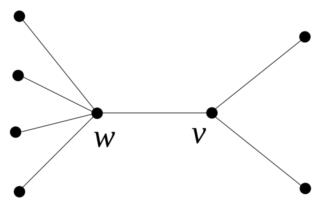

Figure 1: An example of non-optimality of greedy broadcasting algorithm.

In what follows we shall consider only *greedy* broadcasting algorithm which always selects $E_i^* = \partial V_i$. Although not optimal in our model[1], it is appealing due to its simplicity and uniformity.

The main issue in our model is the adversary. With different restrictions on the set F_i, different models are obtained: A *static* adversary has a fixed set of faulty

[1] Consider the graph from Figure 1 and let the broadcast start from vertex v. Let us suppose that the adversary blocks at most one half of active edges in each step. Then the broadcasting time of the greedy algorithm is 5. However, an algorithm which in the fist step tries to send a single message from v to w (this message cannot be blocked by the adversary), yields a broadcasting time 4.

edges $F = F_i$ for all i, i.e. the broadcasting is performed on a fixed subgraph of G. A *dynamic* adversary, introduced in [13], has a fixed number f of faulty edges, i.e. $|F_i| = f$ for all i.

In our approach we shall deal with *proportional* adversary which has a fixed ratio α of faulty edges:

Definition 2 *An adversary for which $|F_i| \leq \alpha|E_i^*|$ for a constant α is called α-proportional.*

In the sequel we shall always consider $1/2$-proportional adversaries. Our main attention is focused to the *broadcasting time*:

Definition 3 *Broadcasting time for an algorithm on a given graph is the largest broadcasting time over all admissible (i.e. proportional) adversaries.*

Unless explicitly stated, all logarithms are to the base 2. By $diam_G$ we denote the diameter of G.

3 Results

We are particularly interested in the impact of structural properties of the underlying topology on the broadcasting time of faulty broadcast with proportional adversary. The main question we ask is: *"In which graphs the adversary cannot asymptotically slow down the broadcasting process?"*, i.e. in which graphs the broadcasting time is $O(diam_G)$. We first show examples of graphs where the broadcasting time is $\omega(diam_G)$. We start with complete graphs K_n:

Theorem 1 *The broadcasting time on K_n is at least $\lfloor \log(n+1) \rfloor - 1$ and at most $\lfloor \log(n-1) \rfloor + 1$.*

Proof. Consider a broadcast $\{w\} = V_0 \subseteq V_1 \subseteq \cdots \subseteq V_t = V$ and let $|V_i| = a_i$. With a_i informed vertices in step i, there are $a_i(n-a_i)$ active edges.

For the upper bound note that the adversary may block at most $\left\lfloor \frac{a_i(n-a_i)}{2} \right\rfloor$ of them and the rest delivers their messages to some uninformed vertices. However, only a_i of these messages may be destined to one particular vertex and it follows that a_{i+1} is at least $a_i + \left\lceil \frac{n-a_i}{2} \right\rceil$.

On the other hand, there is a strategy for the adversary to obtain equality $a_{i+1} = a_i + \left\lceil \frac{n-a_i}{2} \right\rceil$ – just block the edges in such a way as to inform as few vertices as possible.

So we have

$$\frac{n+a_i}{2} \leq a_{i+1} = a_i + \left\lceil \frac{n-a_i}{2} \right\rceil \leq \frac{n+a_i}{2} + 1$$

The solutions of the recurrences

$$p_0 = 1 \qquad\qquad q_0 = 1$$
$$p_{i+1} = \tfrac{n+p_i}{2} \qquad\qquad q_{i+1} = \tfrac{n+q_i}{2} + 1$$

are $p_i = n - \frac{n-1}{2^i}$ and $q_i = n + 2 - \frac{n+1}{2^i}$, respectively, yielding the following bounds for the number of informed vertices after i steps:

$$n - \frac{n-1}{2^i} \le a_i \le n + 2 - \frac{n+1}{2^i}.$$

From this the result follows by noting that the broadcast stops in step i satisfying $n - 1 < a_i \le n$. □

Another example of graphs where the broadcasting may be slowed down asymptotically are complete d-ary trees $T_n^{(d)}$ of height n where the broadcast is started from the root r.

We first consider the case for $d = 2$, i.e. complete binary trees, and then show how to generalize this result to arbitrary d.

Theorem 2 *The broadcasting time for a complete binary tree $T_n^{(2)}$ is at least $\frac{3}{2-\sqrt{2}} 2^{\frac{n}{2}} - O(n)$.*

Proof. The aim of the proof is to present a strategy for the adversary and to analyze its performance. The strategy is "conservative" in a sense that whenever the adversary chooses to block an edge in a given step, he tries to keep this edge blocked in all subsequent steps until there is no other choice. The start of a broadcast is depicted on Figure 2: in time step 0 only the root is informed. In the first step, there are two active edges and the adversary blocks one of them, so there is one vertex which gets informed in step 1 and in all subsequent steps the adversary tries to block the left son of the root. Hence in step 2, with three active edges, there are two vertices which receive the information. The situation in the next step is as follows: there are two trees of height $n - 2$ and one edge which must be blocked by all means.

In order to present the general adversary strategy, let us consider a situation during the broadcast in which there are l complete binary trees $T_n^{(2)}$ where the l roots are informed. Moreover, there are $k \le l$ edges outside these trees (called "external") which must be blocked whenever possible. It may be viewed so that the broadcasting is done on the l trees $T_n^{(2)}$ but the rules for the adversary are altered by a parameter k such that, instead of blocking at most $|E_i^*|/2$ active edges, the adversary may block at most $\left\lfloor \frac{|E_i^*|+k}{2} \right\rfloor - k$ edges.

The strategy for the adversary is as follows: initially, there are $2l \ge 2k$ active edges. One step is taken by blocking only the k external edges, resulting in a situation with $4l$ active edges and k external active edges (Figure 3). From the

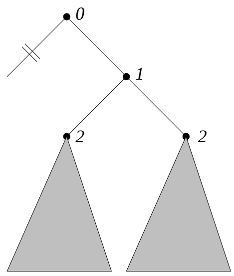

Figure 2: First steps of broadcast on a binary tree.

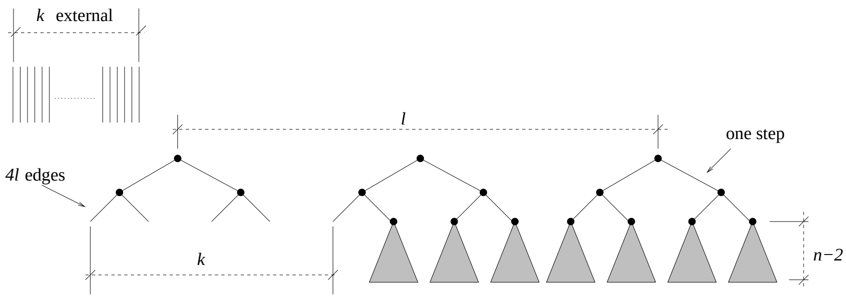

Figure 3: Adversary strategy in binary trees.

$4l$ active edges the adversary blocks the first k, letting the information flow into the roots of $4l-k$ trees of height $n-2$. The same game is now played with $2k$ external edges, $4l-k$ trees and height $n-2$. When this broadcast is finished, the adversary releases the k blocked edges, yielding a situation with k external edges and k trees of height $n-2$. Note that in both recursive calls the condition $l \geq k$ is preserved. If $T(k,l,n)$ denotes the broadcasting time on l trees of height n with k virtual edges, then it holds

$$T(k,l,n) = 3 + T(2k, 4l-k, n-2) + T(k,k,n-2), \text{ for } n \geq 2$$

Now let's compute the term $T(k,l,n)$. As it does not depend on k,l it holds

$$T(k,l,n) \geq T(n) = 3\sum_{i=0}^{h-1} 2^i = 3 \cdot 2^h - 3, \text{ where } h = \lfloor n/2 \rfloor$$

The whole adversary strategy on a tree of height n is as follows: first block one of the two active edges and use the above strategy for $k=1$, $l=1$ and height $n-1$. Then let the message spread along the single blocked edge leaving an analogous situation for a tree of height $n-1$. The resulting time is thus:

$$T'(n) = \sum_{i=1}^{n} 2 + T(n-i)$$

So we get

$$T'(n) \geq 3\sum_{i=1}^{n-1} 2^{\frac{n-i-1}{2}} - n + 2$$

Noting that $\sum_{i=1}^{n-1} 2^{\frac{n-i-1}{2}} = \frac{2^{\frac{n}{2}}-\sqrt{2}}{2-\sqrt{2}}$ yields the result. □

This strategy is easily generalized to d-ary trees in the following way:

Theorem 3 *Consider a complete d-ary tree $T_n^{(d)}$. The broadcasting time is at least* $\Omega\left(\frac{n}{d} + \frac{(d\log d - c)^{\frac{n}{d}}}{d}\right)$ *where* $c = \log 3 - 1$.

Proof. In a similar way to the strategy in Theorem 2 we first describe a strategy on a forest of l trees with $k < l$ external edges. The adversary first spends $d-1$ steps by blocking only the k external edges; at the end there are $l \cdot d^d$ edges. Let $z = \left\lfloor \log \frac{d^d}{3} \right\rfloor$ and divide the $l \cdot d^d$ edges into $z+2$ groups $E_0, \ldots, E_z, E_r$, such that the group E_i, $0 \leq i \leq z$ contains $k \cdot 2^i$ edges and E_r contains the rest, i.e. $l \cdot d^d - k \cdot 2^z$ edges. Next, the adversary plays the game with each of these groups in turn (with the appropriate number of external edges), so we get

$$T(k,l,n) = d + T(k2^{z+1}, ld^d - k2^{z+1}, n-d) + \sum_{i=0}^{z} \left(1 + T(k2^i, (k2^i, n-d)\right)$$

Again, this term does not depend on k,l so we get a bound

$$T(n) = (d+z+1)\frac{(z+2)^{\lfloor \frac{n}{d} \rfloor} - 1}{z+1}$$

The main strategy repeats the above described procedure successively with $\lfloor d/2 \rfloor$, $\lfloor d/4 \rfloor, \ldots, 1$ external edges before descending to lower height, i.e.

$$T'(n) = \lfloor \log d \rfloor \cdot (1 + T(n-1)) + 1 + T'(n-1).$$

After substituting $T(n-1)$, solving and simplifying, one gets the following: $T'(n) = n(1 + \lfloor \log d \rfloor) + \frac{\lfloor \log d \rfloor (d+z+1)}{z+1} \cdot \left[\frac{(z+2)^{\frac{n}{d}} - 1}{(z+2)^{\frac{d-1}{d}} \left[(z+2)^{\frac{1}{d}} - 1 \right]} - n \right]$, and noting that $z + 2 \geq d\log d - c$ yields the result. □

The rest of this section is devoted to the examples of graphs in which the adversary cannot slow down the broadcasting asymptotically. We show that even grids with constant dimension are such graphs.

By an n-dimensional even toroidal grid we mean a graph with vertices $\mathbb{Z}_k^n$, where $k \in 2\mathbb{N}_+ \cup \{\infty\}$ is the length of the grid and an edge connects every pair vertices which differ exactly in one position and exactly by one (in the group $\mathbb{Z}_k$). Let us adopt the following notation. For a set $S \subseteq V$ denote by $S^{(r)}$ the r-neighborhood, i.e. $S^{(r)} = \{x \in V \mid dist(x,S) \leq r\}$ and denote by ∇S the vertex boundary, i.e. $\nabla S = S^{(1)}/S$. For a vertex v denote by $\mathcal{B}_r(v)$ the ball with radius r centered at v, i.e. $\mathcal{B}_r(v) = \{v\}^{(r)}$.

In the sequel we shall use the following isoperimetric inequality due to Bollobás and Leader:

Theorem 4 *[1] Let k be even and let A be a subset of $\mathbb{Z}_k^n$ with*

$$|A| = |\mathcal{B}_r(\mathbf{0})| + \alpha|\nabla\mathcal{B}_r(\mathbf{0})|,$$

where $0 \leq \alpha < 1$. Then

$$|A^{(1)}| \geq |\mathcal{B}_{r+1}(\mathbf{0})| + \alpha|\nabla\mathcal{B}_{r+1}(\mathbf{0})|.$$

Clearly, this theorem holds also for infinite grids. Denote $B_r = |\mathcal{B}_r(\mathbf{0})|$ and $\nabla B_r = |\nabla\mathcal{B}_r(\mathbf{0})|$. Now let us consider a broadcast $\{w\} = V_0 \subseteq V_1 \subseteq \cdots$ in an n-dimensional grid. The following lemma shows the limitation of the adversary: regardless of his strategy, the number of informed vertices increases.

Lemma 1 *Let $|V_i| = B_r + \alpha_i \nabla B_r$ for some r, $0 \leq \alpha_i < 1$. Then $|V_{i+1}| = B_r + \alpha_{i+1}\nabla B_r$ such that*

$$\alpha_{i+1} \geq \frac{\alpha_i}{2n+1}\left(2n + \frac{\nabla B_{r+1}}{\nabla B_r}\right) + \frac{1}{2n+1}.$$

Proof. Suppose that in the i-th step the adversary blocks z vertices from ∇V_i. Clearly, there must be at least one edge for each of them in ∂V_i. As the adversary may block at most $1/2|\partial V_i|$ edges, there must be another at least z edges in ∂V_i leading to some vertices in ∇V_i which are not blocked in i-th step. Because the degree of a vertex is $2n$ it follows that $|\nabla V_i| \geq z + \frac{z}{2n}$ and hence $z \leq |\nabla V_i|\frac{2n}{2n+1}$. The remaining vertices from ∇V_i are part of V_{i+1} and so $|V_{i+1}| \geq |V_i| + 1/(2n+1)|\nabla V_i|$. Using that $\nabla V_i = V_i^{(1)}/V_i$ and $\nabla B_r = B_{r+1} - B_r$ and applying Theorem 4 we get

$$|V_{i+1}| \geq B_r + \alpha_i \nabla B_r + \frac{1}{2n+1}\left(B_{r+1} + \alpha_i \nabla B_{r+1} - B_r - \alpha_i \nabla B_r\right)$$

Regrouping we get that

$$\alpha_{i+1} \geq \frac{\alpha_i \nabla B_r + \frac{1}{2n+1}\left((1-\alpha_i)\nabla B_r + \alpha_i \nabla B_{r+1}\right)}{\nabla B_r}$$

The result follows immediately. □

Now we need to bound the term $\nabla B_{r+1}/\nabla B_r$ by a positive constant in order to show that the adversary may slow down the broadcasting only by a constant factor. In infinite grids it is immediate:

Theorem 5 *Consider a faulty broadcast in an infinite grid of constant dimension n. Then in each step r the number of informed vertices is* $\Theta(B_r)$.

Proof. First note that in an infinite grid it holds $\nabla B_{r+1} > \nabla B_r$ for all r and hence by Lemma 1 $\alpha_{i+1} \geq \alpha_i + 1/(2n+1)$. This means that in any faulty broadcast it holds $|V_{(2n+1)r}| \geq B_r$ for all r. Since B_r is a polynomial in r of degree n, the fraction $B_{(2n+1)r}/B_r$ converges to a constant (depending on n). □

On the other hand, we show a strategy for the adversary to slow down the broadcast in an infinite grid in such a way that $|V_r| \approx B_r/2^n$. The aim of the adversary is to steer the broadcast in such a way that $V_r = \{\langle a_1,\dots,a_n\rangle \mid \sum a_i \leq r,\ a_i \geq 0\}$, which we call a "positive quadrant". From that the result follows, as $B_r = |\mathcal{B}_r(\mathbf{0})| = |\{\langle a_1,\dots,a_n\rangle \mid \sum |a_i| \leq r\}|$.

Now it is sufficient to show that it is always possible for the adversary to block all edges going away from the positive quadrant. We show that for each such edge there is another edge leaving V_r which goes into the positive quadrant. The outgoing edges are of the form $\langle a_1,\dots,0,\dots,a_n\rangle \mapsto \langle a_1,\dots,-1,\dots,a_n\rangle$. The associated edge will be $\langle a_1,\dots,x,\dots,a_n\rangle \mapsto \langle a_1,\dots,x+1,\dots,a_n\rangle$ where x is the largest possible.

For the upper bound on the broadcasting time on finite grids some more reasoning is necessary:

Theorem 6 *In an n-dimensional grid with constant n and even k the broadcasting time is* $\Theta(k)$.

Proof. Let $k = 2l$. We are going to investigate the term $\nabla B_{r+1}/\nabla B_r$ needed in Lemma 1. If $r < l-1$ the situation is identical to that in an infinite grid, i.e. $\nabla B_{r+1} > \nabla B_r$. If $r = l-1$, the "corner" vertices of ∇B_{r+1} (i.e. those $\langle a_1,\dots,a_n\rangle$ for which only one a_i is nonzero) are "cut off". As there are $2n$ corners, it holds $\nabla B_{r+1} > \nabla B_r - 2n$.

The remaining case is when $r > l-1$. In this case both $\nabla\mathcal{B}_{r+1}(\mathbf{0})$ and $\nabla\mathcal{B}_r(\mathbf{0})$ consist of 2^n disjoint parts located in the 2^n "corners" (relative to the center) of the grid. So we get $\nabla B_{r+1} = 2^n Q^n_{s-1}$ and $\nabla B_r = 2^n Q^n_s$ where $s = 2l-r-1$ and $Q^n_s = |\mathcal{Q}^n_s|$, where $\mathcal{Q}^n_s = \{\langle a_1,\dots,a_n\rangle \mid \sum_{i=1}^n a_i = s,\ a_i \geq 0\}$.

We start by noting a recurrence governing the quantities Q^n_s:

$$
\begin{aligned}
Q^1_s &= 1\\
Q^n_1 &= n\\
Q^n_{s+1} &= Q^n_s + Q^{n-1}_{s+1}
\end{aligned}
$$

The last line is due to the following bijection: Consider $\langle a_1,\ldots,a_n\rangle \in Q^n_{s+1}$. If $a_1 > 0$ then $\langle a_1-1,a_2,\ldots,a_n\rangle \in Q^n_s$ and if $a_1 = 0$ then $\langle a_2,\ldots,a_n\rangle \in Q^{n-1}_{s+1}$. It is easy to see that

$$Q^n_s = \binom{s+n-1}{n-1}.$$

Since $s > 0$ we get

$$\frac{\nabla B_{r+1}}{\nabla B_r} = \frac{2^n Q^n_{s-1}}{2^n Q^n_s} = \frac{\binom{s+n-2}{n-1}}{\binom{s+n-1}{n-1}} = \frac{s}{s+n-1} \geq \frac{1}{n}.$$

Now reconsider Lemma 1. The inequality becomes

$$\alpha_{i+1} \geq \frac{\alpha_i}{2n+1}\left(2n+\frac{1}{n}\right)+\frac{1}{2n+1}.$$

Solving this recurrence (with boundary condition $\alpha_0 = 0$) yields

$$\alpha_i = \frac{n}{n-1}\left(1-\left(\frac{2n^2+1}{2n^2+n}\right)^i\right)$$

Solving for $\alpha_i = 1$ we get

$$i = \frac{\log n}{\log(2n^2+n)-\log(2n^2+1)}$$

Lemma 1 thus states that if there are at least B_r informed vertices, after i steps there will be at least B_{r+1} informed vertices, hence the broadcasting time is at most ik. Because n is constant the result follows. □

4 Conclusions and further research

We proposed a variant model of synchronous networks with dynamic link faults and studied the broadcasting time in this model. We showed that grids of constant dimension are fault-resilient for broadcast, i.e. broadcasting may be completed in $O(diam_G)$ time where $diam_G$ is the diameter of the grid. We also proved that complete d-ary trees and cliques are highly non-fault-resilient for broadcast in the proposed model. We plan to extend the results to the cases of proportional adversary with $\alpha \neq 1/2$.

Further research may involve a multitude of questions. It would be interesting to find general criteria implying fault-resilience for broadcasting in the proposed model. The question of fault-resilience is open for many particular topologies: e.g. it follows from the proof of Theorem 6 that the broadcasting time for hypercubes Q_n is at most $O(n^3)$; we lack, however, any non-trivial lower bound.

Another question of interest concerns the complexity of determining, for a given graph G and a vertex v, the broadcasting time of G starting from v.

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